MODELING AND SIMULATION OF MHD CONVECTIVE HEAT TRANSFER OF CHANNEL FLOW HAVING A CAVITY

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ABSTRACT
In this article, mathematical modeling is executed to examine MHD combined convection flow in an open channel with an enclosure which has a heated wall on left side to simulate assisting flow employing finite element method (FEM). Cold fluid flows through a channel which has an open cavity. All the parts of the cavity are adiabatic while the left wall is heated with an isothermal heater. Two enclosures are tested as square (Case 1) and rectangular (Case 2). Inside the channel flow and temperature field of magnetic effects is examined. Furthermore, simulations are conducted for Rayleigh numbers \(10^4 \leq Ra \leq 10^5\) and Hartmann numbers \(0 \leq Ha \leq 100\). For the above parameters streamlines, isotherms, average Nusselt number and average fluid temperature at the exit port, pressure and temperature gradient in the domain results are discussed. In this work heat transfer decreased by 48% and 52% as \(Ha\) increases from 0 to 100 at \(Ra =10^5\) for case 1 and case 2, respectively.

Keywords: Channel, Open cavity, Magnetic field, FEM.

Nomenclature
- \(B_0\): magnetic induction (Tesla)
- \(g\): gravitational acceleration (ms\(^{-2}\))
- \(H\): height of the cavity (m)
- \(Ha\): Hartmann number
- \(L\): length of the cavity (m)
- \(Nu\): average Nusselt number
- \(p\): dimensional pressure (kglm\(^{-1}\)s\(^{-2}\))
- \(Pr\): Prandtl number
- \(Ra\): Rayleigh number
- \(Re\): Reynolds number
- \(T\): dimensional temperature (K)
- \(u, v\): velocity components (ms\(^{-1}\))
- \(U, V\): dimensionless velocity components
- \(x, y\): Cartesian coordinates (m)
- \(X, Y\): dimensionless coordinates

Greek symbols
- \(\alpha\): thermal diffusivity (m\(^2\)s\(^{-1}\))
- \(\beta\): thermal expansion coefficient (K\(^{-1}\))
- \(\gamma\): general dependent variable
- \(\mu\): dynamic viscosity (kg m\(^{-1}\)s\(^{-1}\))
- \(v\): kinematic viscosity (m\(^2\)s\(^{-1}\))
- \(\theta\): non-dimensional temperature

\(\rho\): density (kg m\(^{-3}\))
\(\sigma\): electrical conductivity (Sm\(^{-1}\))
\(\psi\): stream function

Subscripts
- \(h\): heated wall
- \(i\): inlet state

1. INTRODUCTION

High heat generating electric equipments is a common phenomenon. To get the better performances of the components, an effective heat removal process is required for this phenomenon. Fluid flow and heat transfer are investigated numerically and experimentally by many researchers (Incropera, 1988; Peterson and Ortega, 1990). From the last decade of twenty century mixed convection heat transfer is a popular topic in the electric industry. Their main targets were to assume the different cooling process and a reliable cooling method due to the heat reduction of electronic device for different type of geometry. Flush mounted heaters play a vital role in thermal engineering such as furnaces or ovens, cooling of electronical systems, building heating or cooling, radiator and gas heater (Rahimi and Tajbakhsh, 2011).

Partial heaters are used mostly to cooling of electronic equipment via natural convection. The natural convection is investigated by Chu et al. (1976) for heater with the wall of enclosure. Afterward, this work is extended for triangular air filled cavity by inserting a flush-mounted heater (Varol et al., 2006; Deng et al., 2002; Liu and Phan-Thien, 2000; Sankar et al., 2011) tested the effects of different parameters of separate heaters on ordinary convection in enclosures.

In electronic equipments the mixed convection heat transfer in an open channel is investigated by many researchers (Young and Vafai, 1998; Da Silva et al., 2005; Dogan et al., 2006). An open channel with top upright enclosure for pure natural convection (Hasnaoui et al., 1990; Chang et al., 2005; El Alami et al., 2005) or mixed convection (Fusegi, 1997; Khanafer and Vafai, 2000; Aminossadati and Ghasemi, 2009) has also been studied by several authors to investigate the heat transfer mechanism.
Magneto-hydrodynamics (MHD) is defined when the dynamics of electrically conducting fluids in the presence of electromagnetic field is discussed in the science discipline. In the natural process and many engineering applications like as liquid-metal cooling of nuclear reactors and electromagnetic casting, etc MHD plays an important role. So it is considered as a modern task for its engineering application. A numerical investigation on a conjugated effect of joule heating and MHD on double-diffusive mixed convection in a horizontal channel with an open cavity is investigated by Rahman et al. (2011a). Also Galerkin weighted residual method for the numerical simulation is calculated by Rahman et al. (2011b) in MHD mixed convection in a horizontal channel with an open cavity. Inside the cavity the authors presented a noteworthy result for the mentioned parameters on the flow and thermal fields. Oztop (2011) investigated using the finite volume method on influence of exit opening location on mixed convection in a channel with volumetric heat sources. Using Control Volume Finite-Element Method Abbassi and Nassrallah (2007) analyzed numerically the MHD flow and heat transfer in a backward-facing step. They showed heat transfer of fluids of high Prandtl number is notably improved by the magnetic field. MHD mixed convection heat transfer enhancement in a double lid-driven obstructed enclosure is investigated by Billah et al. (2011). Also MHD mixed convection in a lid-driven cavity along with joule heating and a centred heat conducting circular block for the the effect of Reynolds and prandtl numbers is performed by Rahman et al. (2010). Surface irregularity of the geometry has a great important. The collective convection flow through disposed rectangular enclosure with a downhill curly hot top surface is analysed by Hussain (2010). Hussein and Hussain (2010) investigated the effect of hot wavy wall on mixed convection through a lid-driven air-filled square cavity. The mixed convection in trapezoidal cavity with a moving lid is investigated by Mamun et al. (2010). Azwadi and Idris (2010) analyzed ordinary convection in a square cavity using finite different and lattice Boltzmann modelling.

![Figure 1 Physical model of the problem.](image)

From the above discussion, it can be said that the effect of MHD mixed convection on fluid flow and heat transfer has received a considerable concentration in different type of channel and enclosures in current years. But in the literature established small concentration for the the effect of MHD mixed convection on fluid flow and heat transfer in a channel with an open cavity having a heater on the side wall of the cavity. So the heat transfer management by considering the relations between natural and forced convection flows of such engineering problem is required to get the better performances of the electronic device. For this reason in

![Figure 2 Grid independency study: average Nusselt number at different grid elements for Ha = 10.](image)
this study an open cavity having heater on the side wall of the cavity on MHD mixed convection heat transfer in a two-dimensional horizontal channel is considered. The characteristic ratio of the cavity and the effects of magnetic force in the cavity are the major emphasis is given. Finally we may conclude that it helps to design a perfect placement of the high heat generating electronic components and enhances the cooling performance of the electric appliance.

2. PROBLEM DEFINITIONS
In this present work the physical model is described as a two-dimensional horizontal channel with an open cavity. In Figure 1 Physical model is presented as a square shape (Case 1) and rectangular shape (Case 2) enclosure, respectively. \( u_i \) and \( T_i \) are denoted as a uniform air velocity and temperature respectively in the channel. Here laminar and incompressible fluid flow is taken with the Prandtl number of \( \text{Pr} = 7.1 \). The walls of the channel and the bottom and right surfaces of the cavity are insulated. The heater is placed on the left side as full height of the cavity. \( H \) and \( L \) are indicated the height and length of cavity respectively. In addition, channel length is chosen as three times of height of the cavity. Gravity effects in a vertical direction. It means that both forced and natural convections are effective depends on the domination of the effective forces. Magnetic force is induced in the horizontal direction to the open cavity.

3. GOVERNING EQUATIONS WITH BOUNDARY CONDITIONS
Figure 1 exposed an electrically conducting fluid is assumed in two-dimensional, laminar, steady mixed convection flow within a horizontal channel in the existence of a magnetic field appeared opposing to the flow path. In addition, the fluid is taken as a Newtonian fluid with constant properties except the density in the buoyancy term in the momentum equation. The non-dimensional governing equations are as follows:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]  

(1)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]  

(2)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\text{Ra}}{\text{Re}^2 \text{Pr}} \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}
\]  

(3)

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{\text{Re} \text{Pr}} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]  

(4)

Using the depth of the cavity \( \left( X = \frac{X}{H}, Y = \frac{Y}{H} \right) \), the inlet velocity \( \left( U, \frac{U}{u_i} \right), V = \left( \frac{V}{u_i} \right) \), \( \frac{p}{\rho u_i^2} \) and \( \theta = (T - T_i) / (\beta H) \) all the lengths, velocities, pressure and temperature are normalized respectively. The non-dimensional parameters used in equations (1)–(4) are defined as:

Rayleigh number, \( \text{Ra} = \frac{\beta (T_h - T_i) H^3}{\nu \alpha} \), and Hartmann number, \( \text{Ha} = \frac{\sigma B H^2}{\nu} \).

No-slip boundary conditions (\( U = V = 0 \)) are taken on all the walls for fully-developed conditions. \( U = 1, V = 0 \) and \( \frac{\partial U}{\partial X} = 0, V = 0 \) are the velocity conditions at the entry section and at the exit section respectively. Furthermore, \( \theta = 0 \) is considered as the inlet temperature, \( \frac{\partial \theta}{\partial N} = 0 \) at all solid boundaries except heated wall and \( \theta = 1 \) moreover at the heated wall the temperature of the cavity.

\[ N_a = -\frac{1}{2} \int (\frac{\partial \theta}{\partial Y}) \]  

(5)

Using the Eq. (5) the momentum equations (2) - (4), where the pressure \( P \) is terminated with a penalty constraint \( \xi \).

Using the Eq. (5) the momentum equations (2) - (4) can be written as:

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{1}{\text{Re}} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) + \frac{\text{Ra}}{\text{Re}^2 \text{Pr}} \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}
\]  

(6)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{1}{\text{Re}} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + \frac{\text{Ra}}{\text{Re}^2 \text{Pr}} \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}
\]  

(7)

Introducing the velocity components \( (U, V) \) and temperature (\( \theta \)) using basis set \( \phi_k \) \( \theta_k \) as:

\[
U = \sum_{k=1}^{N} U_k \phi_k (X,Y), V = \sum_{k=1}^{N} V_k \theta_k (X,Y) \quad \text{and} \quad \theta = \sum_{k=1}^{N} \theta_k \phi_k (X,Y)
\]  

(8)

Eqs. (4), (6) and (7), respectively at nodes of internal domain \( A \):
Eqs. (4), (6) and (7) at nodes of internal domain A gives the following nonlinear residual equations respectively using the Galerkin finite element method:

\[ R^1_k = \sum_{k=1}^{N} \theta_k \int_{A} \left[ \sum_{k=1}^{N} U_k \frac{\partial \phi_k}{\partial X} + \sum_{k=1}^{N} V_k \frac{\partial \phi_k}{\partial Y} \right] \phi_k dXdY \]

\[ - \frac{1}{RePr} \sum_{k=1}^{N} \theta_k \int_{A} \left[ \frac{\partial \phi_k}{\partial X} + \frac{\partial \phi_k}{\partial Y} \right] dXdY \] (9)

\[ R^2_k = \sum_{k=1}^{N} U_k \int_{A} \left[ \sum_{k=1}^{N} U_k \frac{\partial \phi_k}{\partial X} + \sum_{k=1}^{N} V_k \frac{\partial \phi_k}{\partial Y} \right] \phi_k dXdY \]

\[ - \gamma \sum_{k=1}^{N} U_k \int_{A} \left[ \frac{\partial \phi_k}{\partial X} \frac{\partial \phi_k}{\partial X} + \frac{\partial \phi_k}{\partial Y} \frac{\partial \phi_k}{\partial Y} \right] dXdY \]

\[ - \frac{1}{Re} \sum_{k=1}^{N} U_k \int_{A} \left[ \frac{\partial \phi_k}{\partial X} + \frac{\partial \phi_k}{\partial Y} \right] dXdY \]

\[ \int_{A} \left[ \sum_{k=1}^{N} \theta_k \phi_k \right] \phi_k dXdY + \frac{Ha^2}{Re} \int_{A} \left[ \sum_{k=1}^{N} V_k \phi_k \right] \phi_k dXdY \] (10)

\[ R^3_k = \sum_{k=1}^{N} V_k \int_{A} \left[ \sum_{k=1}^{N} U_k \frac{\partial \phi_k}{\partial X} + \sum_{k=1}^{N} V_k \frac{\partial \phi_k}{\partial Y} \right] \phi_k dXdY \]

\[ \int_{A} \left[ \sum_{k=1}^{N} \theta_k \phi_k \right] \phi_k dXdY + \frac{Ha^2}{Re} \int_{A} \left[ \sum_{k=1}^{N} V_k \phi_k \right] \phi_k dXdY \] (11)

Three points Gaussian quadrature is performed to evaluate the integrals in the residual equations. Later on by the help of Newton–Raphson method the non-linear residual aforementioned equations (9 – 11) are solved to compute the coefficients of the term in Eq. (8). Iteratively the solution is solved until the succeeding convergence form is satisfied: \[ |Y^{m+1} - Y^m| < 10^{-5} \] where \( m \) stands for number of iteration and \( Y \) stands for the general dependent variable.

Table 1 Comparison of results for validation at \( Pr = 0.71, Re = 100, Ri = 0.1, w/H = 0.5, L/H = 2 \).

<table>
<thead>
<tr>
<th>Opposing flow</th>
<th>Present</th>
<th>Manca et al. (2003)</th>
<th>% of error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nu</td>
<td>1.7657</td>
<td>1.7748</td>
<td>0.52%</td>
</tr>
<tr>
<td>( \theta_{max} )</td>
<td>0.629</td>
<td>0.627</td>
<td>0.31%</td>
</tr>
</tbody>
</table>

Grid independency study is depicted in Figure 2 for different Rayleigh number. Five grid dimensions as 3536, 5120, 6248, 8106 and 9636 are tested. These test have done for \( Ha = 10 \). 5120 elements as grid dimension is chosen for whole tests.

The computer code is validated with previous study of Manca et al. (2003) by supplying same flow and thermal conditions. Results are computed in Table 1 for \( Pr = 0.71, Re = 100, Ri = 0.1, w/H = 0.5, L/H = 2 \). Average Nusselt number and maximum temperature values are compared. The result from the Table 1, shows good agreement with literature results.

### 5. RESULTS AND DISCUSSION

Application of finite-element method was executed to see the results of magnetic force conditions on combined convection heat transfer and fluid flow. Two cases were performed according to cavity aspect ratio as square (Case 1) and rectangular (Case 2) cavity. In both cases assisting a flow is modeled under magnetic forces. The implications of varying the Rayleigh numbers \( Ra \), Hartmann number \( Ha \) and physical parameters for the scheme are the cavity aspect ratio \( AR (= L/H) \) will be emphasized. For the three different regimes of flow \( Ra = 10^2, 10^3 \) and \( 10^4 \) of streamline and isotherm patterns are presented here. The variations of the average Nusselt number at the heated surface, average fluid temperature at the exit port and pressure and temperature gradient in the domain for the different values of the parameters were calculated.

![Figure 3](image3.png)

Figure 3 (a) Streamlines and (b) Isotherms for the case 1 at \( Ra = 10^3 \), and selected values of Hartmann number \( Ha \).
5.1 Flow and thermal fields

5.1.1 Case 1

Effect of Hartmann number for the streamlines (on the left) and isotherms (on the right) at Ra = 10^3 is shown in Figure 3. It is very clearly from the figures, lower value of Rayleigh number, flow distribution of the cavity is not so effective for heating part and inside the cavity there is no any circulation when inlet flow goes through the channel from the top wall of the channel except Ha = 0. One may notice that in absence of magnetic field a rotating cell is created in the clockwise direction and ψmin = -0.001 at the bottom part of the cavity. Left heated vertical wall scattered isotherms into the cavity and from right top side of the cavity hot fluid put down.

Figure 4 shows the effect of Hartmann number on the flow field and temperature fields at Ra = 10^4. We observe that the flow and temperature fields are almost same as Figure 3 for higher values Hartmann number Ha (= 50 and 100). But, an interesting result is found that a rotating cell is shaped in the clockwise direction and ψmin = -0.063 in absence of magnetic field.

![Figure 4](image1)

Figure 4 (a) Streamlines and (b) Isotherms for the case 1 at Ra = 10^4, and selected values of Hartmann number Ha.

It is also noticed that the clockwise rotating cell covered most of the part of the cavity. The cavity is like differentially heated cavity from left and right due to heated left wall of the cavity. Curvy circulation is shaped by isotherms which is parallel to left wall. The influence of Hartmann number on the streamlines and isotherms has been displayed in Fig 5 at Ra = 10^5. One may notice that both the flow field as well as the thermal field strongly influenced for higher values of Rayleigh number. In the clockwise direction the rotating cell is created and ψmin = -0.274 in absence of magnetic field. If this figure is compared with Figure 3, the circulation cell becomes stronger with ψmin = -0.274 at Ha = 0. It is also noticed that the clockwise rotating cell occupies almost whole of the part of the cavity. This clockwise circulating cell is decreasing when Hartmann number increasing. The location of the main centre is changed a little bit to right side. The circulating cell centre move to vertical heated. It is also clearly seen that the shape and size of the eddy changes while magnetic force changes. Figure 5 shows that for Ra = 10^5, thermal boundary layer turns into thinner in case of higher values of Rayleigh number. Plume like temperature distribution is seen for Ha = 0. Isotherms are parallel to the heater for Ha = 100 due to low flow velocity. Due to increasing buoyancy effective flow isotherms are clustered around the heater and fluid flows directly over the cavity.

![Figure 5](image2)

Figure 5 (a) Streamlines and (b) Isotherms for the case 1 at Ra = 10^5, and selected values of Hartmann number Ha.
5.1.2 Case 2

Figures 6 - 8 present the outcome of Hartmann number on the streamlines (on the left) and isotherms (on the right). Figure 6 shows the streamline (on the left) and isotherms (on the right) for different values of Hartmann numbers at $Ra = 10^3$. In this cases, heating part of the cavity does not an useful parameter on the flow field and from the top wall of the channel inlet flow goes through without any circulation inside the cavity apart from $Ha = 0$. For $Ha = 0$, a very small circulation cell with clockwise rotating direction is formed at right bottom corner of the cavity due to domination of buoyancy force.

Figure 6 (on the right) illustrates the isotherms to see the effects of temperature distribution with different magnetic forces. Left heated vertical wall into the cavity created isotherms and hot fluid go away from right top side of the cavity. From the figure, thermal boundary layer is converting into thicker with decreasing of Hartmann number. The effect of Hartmann number on the flow field and temperature fields has been shown in Fig 7 at $Ra = 10^4$. It can be seen that the flow and temperature fields are almost identical as Figure 6 for higher values Hartmann number $Ha (= 25, 50$ and $100)$. But, an interesting result is found that three circulating cells are formed in absence of magnetic field. As seen from the figure, thermal layer becomes thicker with decreasing of Hartmann number. For $Ha = 0$, plume like allocation is formed. Figure 8 is plotted streamlines and isotherms for different values of Hartmann number $Ha = 0, 25, 50$ and $100$ at $Ra = 10^5$. The buoyancy-induced clockwise rotating cell of fluid is shown for the lowest value of $Ha = 0$ near the heating wall of the cavity shown from the left column of this figure. The strength of the rotating cell is reduced and pushed to the left bottom corner of the cavity with the increase of Hartmann number. The flow pattern gains strength in the channel and occupies the whole cavity, which shows the conducting mode of heat transfer.

Figure 7 (a) Streamlines and (b) Isotherms for the case 2 at $Ra = 10^4$, and selected values of Hartmann number $Ha$.

A higher value of Hartmann number, which is a measure of magnetic field, retards the flow velocity. Thus, this recirculation cell becomes smaller at $Ha = 50, 100$ and it disappeared for further values of magnetic field. Natural Convective twist appears inside the cavity of the subsequent isotherms for the lowest value of $Ha = 10$. 

Figure 8 (a) Streamlines and (b) Isotherms for the case 2 at $Ra = 10^5$, and selected values of Hartmann number $Ha$. 

(a) 

(b) 

(a) 

(b)
The deformations of isothermal lines showed for the high convective current inside the cavity. When Hartmann number is increasing then deformations of isothermal lines are disappearing. Inside the cavity and the channel isothermal lines move towards more and more near the conduction like circulation pattern of isothermal lines as Hartmann number increases. The convection is approximately restrained and the isotherms are nearly parallel to the horizontal wall for higher value of Hartmann number $Ha = 50$ and $100$. It is indicating that a quasi conduction command reached. Plume like temperature distribution is seen for $Ha = 0$ and $25$.

Figure 8 (a) Streamlines and (b) Isotherms for the case 2 at $Ra = 10^5$, and selected values of Hartmann number $Ha$.

5.2. Heat Transfer
5.2.1 Case 1
At the exit port, Figure 9 (a) and (b) illustrate the average Nusselt number and average fluid temperature respectively. The figures indicate for different Rayleigh numbers at selected values of Hartmann numbers. At the exit port are demonstrated similar movements by both Nusselt number and average fluid temperature. In addition, both heat transfer and average fluid temperature decrease with increasing of Hartmann numbers. Heat transfer turns into constant for $Ra = 10^5$, $10^4$ and $10^3$ due to decreasing of flow velocity with increasing of strength of magnetic field.

Figure 9 (a) Average Nusselt number and (b) average fluid temperature at the exit port versus Rayleigh number $Ra$ for the case 1 and selected values of Hartmann number $Ha$.

Figure 10 shows the pressure and temperature gradient for different Rayleigh number. In addition, it shows that pressure tends to zero up to $Ra = 10^4$ but for the higher values of Rayleigh number it is negative (Figure 10 (a)). From the Figure 10 (b), it is clear general view of temperature gradient show decreasing behaviour for corresponding Rayleigh number. It is noticed that temperature becomes constant for $Ra = 10^3$ and $10^4$ between $Ha = 50$ and $Ha = 100$ as given in Figure 10 (b).

5.2.2 Case 2
Figure 11 (a) and (b), shows the distinction of average Nusselt number and average fluid temperature at the exit port respectively. At the exit port both Nusselt number and average fluid temperature reveals comparable inclinations. Additionally, both heat transfer rate and average fluid temperature decrease with escalating of Hartmann numbers. One may notice that heat transfer rate decreases very smoothly $Ra = 10^5$ with increasing of strength of magnetic field.

On the other hand, Figure 12 presents pressure and temperature gradient in the domain for different Rayleigh number. It is clear that pressure is positive up to $Ra = 10^4$ although for higher values of Rayleigh number it is negative. Also in Figure 12 (b) general view of
Figure 10 (a) Pressure and (b) temperature gradient in the domain versus Rayleigh number $Ra$ for the case 1, and selected values of Hartmann number $Ha$.

Figure 12 (a) Pressure and (b) temperature gradient in the domain versus Rayleigh number $Ra$ for the case 2, and selected values of Hartmann number $Ha$.

A comparison study has depicted in Figure 13. It display the effects of average Nusselt number versus Hartmann number $Ha$ for (a) $Ra = 10^3$, (b) $Ra = 10^4$, and (c) $Ra = 10^5$ for the case 1 and 2. It is clearly observed that heat transfer decreases very slowly as $Ha$ increases from 0 to 100 at lower Rayleigh numbers, $Ra (=10^3$ and $10^4$) for both cases. On the other hand, it is noticed that heat transfer decreases constantly with increasing $Ha$ from 0 to 100 at higher Rayleigh numbers, $Ra (=10^5$) for both cases. However, more heat transfer occurs for the case 2 than case 1 for higher Rayleigh numbers, $Ra (=10^5)$.

6. CONCLUSIONS
Here flow field, heat transfer and temperature distribution in fully heated open ended cavities under magnetic field are computationally analyzed via the finite-element technique for two different cases and Rayleigh numbers. Following conclusions can be taken from the numerical analysis of the results:

- For both cases, flow strength as well as heat transfer increase with Rayleigh number.
- Flow velocity reduces flow strength and heat transfer with increasing of Hartmann number. Thus, magnetic field can be treated as a control parameter for heat
transfer and fluid flow in open ended channel flow with cavity.
- Heat transfer rate is proportional to Rayleigh number for both of the cases.

The thermal design of electronic packages can be made from the recommended guidelines of this study. The geometry of electronic devices and the arrangement of electronic components are of interest.

Figure 13 Average Nusselt number versus Hartmann number \( Ha \) for (a) \( Ra = 10^3 \), (b) \( Ra = 10^4 \), and (c) \( Ra = 10^5 \) for the case 1 and 2.

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