THEORETICAL ANALYSIS OF THERMAL PROFILE AND HEAT TRANSFER IN GRINDING

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ABSTRACT
Temperature in grinding is one of the major factors, affecting the quality of the ground surface. A review of thermal modelling approaches in grinding is presented. During grinding, heat is generated at the wheel-work interface. This problem can be likened to Jaeger’s moving heat source, by analogy. The general solution of moving heat source problem by J.C. Jaeger is applied to the grinding process. Rectangular, triangular and parabolic laws of heat flux are tested to represent the grinding zone (a moving heat source) and the resulting temperature distributions are compared. Triangular heat source predicts the thermal profile with 10% error. The factors influencing the temperature for burn are identified by thermal modelling. A re-look at limiting temperature for burn incidence indicates that limiting value is higher than \(A_c\) temperature for steel. Modelling of the heat partition is helpful to assess alternate ways to control grinding temperature.

Keywords: Grinding temperature, Burn, Partition ratio.

NOMENCLATURE
\(\rho\) -Density (Kg/m\(^3\))
\(\varepsilon\) -Energy Partition ratio
\(\varepsilon_t\) -Energy Partition ratio due to chip formation
\(\alpha\) -thermal diffusivity (m\(^2\)/s)
\(a\) -depth of cut (\(\mu\)m)
\(A\) -Area of point heat source (m\(^2\))
\(A_r\) -austenite finish temperature for steel (\(^\circ\)C)
\(B\) -Constant
\(c\) -Specific heat capacity (J/Kg*\(^\circ\)K)
\(d_e\) -equivalent diameter (m)
\(\gamma\) -Thermal conductivity (W/m*K)
\(K_0(x)\) -Modified Bessel’s function of second kind of order 0
\(I\) -half-length of heat source (m)
\(X, Y, Z\) -Non-dimensional parameters
\(L, U\) -Non-dimensional parameters
\(l_k\) -Wheel-work contact length (m)
\(P_e\) -Peclet number
\(Q\) -Heat instantaneously liberated (J)
\(q\) -Heat liberated per unit time per unit area (W/mm\(^2\))
\(T\) -Temperature of semi-infinite body (\(^\circ\)C)
\(T_i\) -Initial temperature (\(^\circ\)C)
\(t\) -Time (s)
\(T_m\) -Maximum Temperature (\(^\circ\)C)

\(u\) -Specific grinding energy (J/mm\(^3\))
\(u_0\) -Portion of chip formation energy going to chips and coolant (J/mm\(^3\))
\(u_{ch}\) -Specific chip formation energy (J/mm\(^3\))
\(V\) -Velocity of moving heat source (m/s)
\(V_w\) -Work speed (m/s)
\(V_s\) -Wheel speed (m/s)
\(x, y, z\) -Co-ordinates (m)
\(x'y'z'\) -Co-ordinates (m)
\((\gamma pc)_t\) -Wheel thermal properties (conductivity, density, specific heat capacity)
\((\gamma pc)_w\) -Work piece thermal properties (conductivity, density, specific heat capacity)
\((\gamma pc)_c\) -Composite wheel thermal properties (conductivity, density, specific heat capacity)
\(k_g\) -Abrasive grain thermal conductivity (W/m*K)
\(k_c\) -Composite wheel thermal conductivity (W/m*K)
\(k_f\) -Coolant thermal conductivity (W/m*K)
\(r_o\) -Grain contact dimension (wear flat land)
\(\varphi_a\) -near surface porosity of the abrasive wheel in %
\((pc)_g\) volumetric specific heat capacity of grain
\((pc)_h\) volumetric specific heat capacity of fluid
\((pc)_c\) volumetric specific heat capacity of composite wheel

1. INTRODUCTION
Grinding is an abrasive machining process in which an abrasive wheel rotating at a high speed, removes material in the form of small chips from the work piece moving across it. As the grain first contacts the work piece, at the beginning of contact zone, sliding and ploughing of the grain on the work piece may occur. As the grain moves further, it encounters a thick layer of material, force builds up and material is removed in the form of chips. The representative chip formation process in grinding is presented in Figure 1. Heat is generated at the wheel-work interface due to plastic deformation leading to chip formation as well as frictional sliding. As the chips travel along the rake face of the grain, further heat is generated at the contact zone. Thus very high temperature occurs over the cutting edges in the grinding zone, which often necessitates the use of coolant in order to control the temperature rise.

The energy consumed and consequently the temperature at the grinding zone is highly dependent on the nature of
wheel-work interface and hence less amenable for an accurate control. Temperature measuring methods alone may not be adequate to identify and control the grinding process. Hence a mathematical analysis of heat transfer in grinding coupled with on-line monitoring techniques can offer a better approach to control the thermal damage due to high temperature generated in grinding. This article presents the theoretical modelling and analysis of the temperature distribution in a surface grinding process. Further, modelling of peak temperature and heat partition in grinding has been reviewed. The effect of varying strength of heat flux within the grinding zone is also studied. The heating rate at the work surface is estimated to know the burn threshold. Finally, the heat partition phenomenon has been shown to be a decisive factor affecting the thermal profile.

2. THERMAL MODEL OF GRINDING PROCESS

Interest in the thermal models for the grinding process is currently high as manufacturers are expressing concern to avoid thermal damages such as the formation of bluish temper colour due to burn, thermal cracks, residual tensile stresses, etc. Analytical models have been established for the thermal analysis of regular, creep-feed and high efficiency deep grinding practices with specific techniques to account for differences in the process and in the grinding conditions (Malkin and Guo, 2007). The finite element method was employed to develop a three dimensional topology map for the wet grinding temperature distribution (Lin et al., 2009). The inverse heat transfer method was used to estimate the heat source shape, intensity and the grinding temperature variations (Kim et al., 2006; Li et al., 2011; Hua et al., 2011).

In grinding, several grains of the grinding wheel operate simultaneously on the work piece, giving rise to several small moving heat sources of varying intensity. So the work piece can be considered to be subjected to a continuously acting band heat source over the wheel-work interface (DesRuisseaux and Zerkele, 1970). The intensity of heat source may follow a rectangular, triangular or a parabolic law as shown in Figure 2. Hence, the temperature distribution within the work piece was calculated for the case of a strip of heat source moving on the surface of a semi-infinite solid (Carslaw and Jaeger, 1947).

2.1 Modelling the temperature profile

Temperature field in the ground surface can be calculated by modelling the grinding zone with a moving heat source. The moving heat source model has been used widely viz., thermal analysis of welding process (Ozisik, 1993; Darmadi et al., 2011), sliding surfaces (Jaeger, 1942; Bansal and Streator, 2009) and orthogonal cutting (Saoubi and Chandrasekaran, 2011).

In grinding, several grains of the grinding wheel operate simultaneously on the work piece, giving rise to several small moving heat sources of varying intensity. So the work piece can be considered to be subjected to a continuously acting band heat source over the wheel-work interface (DesRuisseaux and Zerkele, 1970). The intensity of heat source may follow a rectangular, triangular or a parabolic law as shown in Figure 2. Hence, the temperature distribution within the work piece was calculated for the case of a strip of heat source moving on the surface of a semi-infinite solid (Carslaw and Jaeger, 1947).

Figure 1 Schematic diagram of surface grinding process.

Figure 2 Typical heat flux distributions.
Carslaw and Jaeger (1947) termed problems of this type as cases of variable temperature and introduced an ingenious approach named the heat source method for solving such problems (Hou and Komanduri, 2000). The temperature solution for a point heat source has been obtained using the Dirac delta function (Kuo and Lin, 2006). The temperature response (one-dimensional) that results from a short, instantaneous pulse of energy at the surface of a semi-infinite solid, having a magnitude of Q/Δm² is (Hollman, 2010),

\[ T - T_i = \frac{Q}{\alpha \Delta m} \exp(-x^2/4at) \]  

(1)

where, \( z \) is depth below the surface. The initial and boundary conditions being taken as \( T = T_i \) for \( t = 0 \) and \( (T - T_i) = 0 \); for \( t = \infty \).

Taking the solution for the temperature in an infinite solid due to an instantaneous point heat source to be fundamental, Carslaw and Jaeger (1947) obtained the two-dimensional solution for an instantaneous line source, parallel to y-axis and through the point \((x', 0, z')\). The temperature rise over \( T_i \) at the point \((x, 0, z)\) when heat is liberated by

\[ T = \frac{Q}{4\pi \alpha y} \exp \left[ -\frac{(x-x')^2 + (z-z')^2}{4at} \right] \]  

(2)

Here Q units of heat are instantaneously liberated per unit length of the line source in the semi-infinite solid. The solution for a moving heat source could be obtained by integrating Eq.(2) over time.

It is assumed that the heat source motion has gone infinitely long so that steady state conditions have been attained. If the centre of the heat source is at the origin at time \( t = 0 \); the temperature at a given instant is to be estimated at the point \((x, 0, z)\) when heat is liberated by the line source at \((x', 0, 0)\). At time \( t' \) earlier, the centre of the heat source was at \((-Vt)\). According to Eq.(2), the temperature at zero time at the point \((x, 0, z)\), due to a heat source \((q \ dx' \ dt)\) heat units per unit length along y-axis, and passing through the point \((x' - Vt, 0)\) is given by the equation,

\[ T = \frac{Q}{4\pi \alpha y} \int_{-\infty}^{\infty} dx' \exp \left[ -\frac{(x-x')^2 + (z-z')^2}{4at} \right] \]  

(3)

In the grinding process, heat influx into the work piece occurs as the work surface passes through the grinding zone. Therefore, this problem is equivalent to that of a strip heat source moving over the surface of the work piece, which is a semi-infinite solid. The strip heat source is parallel to the y-axis and of length equal to the arc length of contact. Heat is liberated by the source at the rate of \( q \) per unit time per unit area. As the work piece is a semi-infinite solid, there is no heat flow above the plane \( z = 0 \) and whole of the heat flows into the region \( z > 0 \) (Figure 2). Hence the resultant solution will be twice of that for the same source moving in an infinite medium.

The temperature in the semi-infinite solid due to a band(strip) heat source of length 2\( l \), moving for an infinite time, can be obtained from Eq.(3), integrating with respect to \( x' \) from \(-l \) to \( l \) and with respect to \( t' \), from 0 to \( \infty \). This equation involves double integration and needs the evaluation of an improper integral.

\[ T = \frac{q}{2\pi \alpha y} \int_{-\infty}^{\infty} dx' \int_{0}^{\infty} dt' \exp \left[ -\frac{(x-x')^2 + (z-z')^2}{4at} \right] \]  

(4)

Substituting \( \xi = \frac{Vt}{2\alpha} \)

\[ T = \frac{q}{2\pi \alpha y} \int_{-\infty}^{\infty} dx' \int_{0}^{\infty} dt' \exp \left[ -\frac{V^2 t'^2}{4\alpha^2} - \frac{2(x-x')Vt}{4\alpha} - \frac{z'^2}{4\xi^2} \right] \]  

(5)

\[ T = \frac{q}{2\pi \alpha y} \int_{-\infty}^{\infty} dx' e^{-\frac{(x-x')^2}{2\alpha^2}} \int_{0}^{\infty} dt' \exp \left[ -\frac{V^2 t'^2}{4\alpha^2} - \frac{2(x-x')Vt}{4\alpha} - \frac{z'^2}{4\xi^2} \right] \]  

(6)

where, \( K_0 \) is the Modified Bessel Function of Second Kind of Order 0.

Introducing the following dimensionless quantities,

\[ X = \frac{Vx}{2\alpha} ; \quad Y = \frac{Vy}{2\alpha} ; \quad Z = \frac{Vz}{2\alpha} ; \]  

\[ L = \frac{Vl}{2\alpha} ; \quad U = \frac{V(x-x')}{2\alpha} \]  

(7)

\[ T = \frac{2q\alpha}{\pi\gamma V} \int_{X-L}^{X+L} e^{-U} \cdot K_0(U^2 + Z^2)^{1/2} \ dU \]

For a heat source of strength \( q \) at \( x' \) the Eq.(6) becomes

\[ T = \frac{q}{\pi \gamma} \int_{-l}^{l} e^{-\frac{(x-x')^2}{2\alpha^2}} \cdot K_0 \left( \frac{V}{2\alpha} \sqrt{(x-x')^2 + z'^2} \right) \cdot f(x') \ dx' \]  

(8)

where, \( f(x') = 1 \) for a constant heat source

\[ f(x') = (x' + 1)l \]  

for a linear law with maximum at the front and zero at the rear.
f(x') = 3(t^2 - x'^2)/2t^2 for a parabolic law.

A computer program was developed and used to solve the closed form integral in Eq.(8). The Modified Bessel’s Function was evaluated using a polynomial approximation (Abramowitz and Stegun, 1964; Press et al., 1992).

2.2 Modelling the peak temperature in grinding

The Jaeger’s solution to the moving heat source problem, discussed in Section 2.1, gives the temperature distribution in the semi-infinite solid over which the heat source moves (Jaeger, 1942), But with regard to the grinding problem, the maximum temperature attained by the work piece is of prime importance. Formulae have been developed to approximate the peak temperature, based on the Jaeger’s two dimensional model (Takazawa, 1966; DesRuisseaux and Zerkele, 1970) and are found helpful in preparing computerized routines for simulation of grinding process (Chiu and Malkin, 1993). The expression for peak temperature, formulated by Takazawa (1966) with a wide field of application is,

\[ T_m \pi N \frac{w}{2q}\alpha = 3.1t^{0.53}\exp\left[-0.69L^{-0.39}Z\right] \]

for 1 < L < 80; 0 < Z < 4 (9)

Grinding temperature can be given by a simpler expression (Malkin, 1978);

\[ T_m \frac{\pi N}{2q\alpha} = 3.543L^{1/2} \]

Where \( L = \frac{Vl}{2\alpha} \)

Now, an expression for the maximum temperature can be obtained by substituting the grinding variables appropriately in the non-dimensional parameter \( L \) of the Eq.(10). The velocity of heat source \( V \), corresponds to work piece velocity \( V_w \) and the heat source length, \( 2l = l_c = (a. d)^{1/2} \). Thus,

\[ T_m = \frac{1.13q\alpha^{1/2}a^{-3/4}d_{e}^{1/4}}{q^{1/2}w^{1/2}} \]

where, the partition ratio (\( \varepsilon \)) signifies that only a fraction of the grinding heat is absorbed by the work piece.

By calorimetric method it was proved that all the grinding energy except for about 45% of chip formation energy is conducted as heat into the work piece (Malkin and Anderson, 1974). Hence, the partition ratio was expressed in terms of specific grinding energy as

\[ \varepsilon = \frac{u - 0.45u_{ch}}{u} = \varepsilon_1 \]

Expressing the heat flux per unit width in terms of specific grinding energy, we get

\[ q = \frac{uV_{w}a}{l_c}, \text{ and the expression for maximum temperature becomes,} \]

\[ T_m = \frac{1.13\alpha^{1/2} a^{3/4}V_{w}^{1/2}(u - 0.45u_{ch})}{q^{1/2}} \]

This indicates the contact length, work speed, and the grinding energy may be treated as key factors for the control of temperature. The grinding energy, constituted by cutting, sliding and ploughing components, reflects the wheel sharpness.

2.3 Modelling the burn incidence

One of the most common thermal damage is the work piece burn, which is accompanied by a colour change and visible marks due to severe plastic deformation at the machined surface. Several on-line techniques are available for prediction and control of work piece burn, including the thermocouple, the acoustic emission and the power monitoring methods. Each has its own limitation in application. For a better control over the thermal damage by burn, the on-line techniques must be supplemented with a theoretical analysis of the thermal phenomenon in order to identify the critical region of working, wherein more caution must be exercised.

Grinding burn is known to be a high temperature phenomenon and therefore accurate estimate of the maximum grinding temperature, as in Section 2.2, is essential for the study. By further simplification of the Eq.(13), the linear dependence of maximum grinding zone temperature on the specific grinding energy is explicit (Malkin, 1974).

\[ u = u_0 + Bd_{e}^{1/4}a^{-3/4}V_{w}^{-1/2} \]

where, \( u_0 = 0.45u_{ch} \) and \( B = (\gamma T_m)/1.13a^{1/2} \)

The quantity \( B \) in Eq.(14), was evaluated through experiments. For steel work piece, \( B=7.2 \text{ J/mm}^2 \text{s}^{1/2} \) at burn threshold. This indicated a maximum grinding temperature of 723°C, close to the \( A_{c1} \) (eutectoid) temperature. So, grinding burn is associated with austenitization of the surface layers of the work piece (Malkin, 1989).

Evidences from literature shows that the \( A_{c1} \) temperature (cause of burn) could be strongly affected by the high heating rates experienced at the work surface. This has been investigated using dilatometry and resistivity techniques for evaluating the \( A_{c1} \) and \( A_{c3} \) temperatures for different steels at various heating rates (Feuerstein and Smith, 1954). Heating speed was referred to study the flash temperatures and martensitic transformations in
grinding (Lefebvre et al., 2008; Littman and Wulff, 1955). The mechanism and kinetics of the reaustenitization process in normalized carbon steels has been elaborated to study the effect of the initial microstructure on the transformational kinetics (Jacot and Rappaz, 1997). They have used computational methods for constructing the reverse TTT diagrams and observed that the initial size of the ferritic domains has a strong influence on the transformation kinetics. For example, changing the ferritic zones from 2 microns to 20 microns increased the transformation temperature from 775°C to 870°C for a heating rate of 10°C/s. At very low heating rates (0.05°C/s), the carbon concentration remains uniform in both the phases during the transformation and the austenitization ends just above A_1 (770°C) whereas, at high heating rates (50°C/s), the transformation ends at 912°C.

Due to the high heating rate and the subsequent cooling rate of the surface layers of the work piece under the moving heat source, extreme temperature may lead to rehardening of the surface layer. This has been proved by metallurgical analysis (Malkin, 1974; Fedoseev and Malkin, 1991). Such defects in grinding are comparable to those of welding where the high heating and rapid cooling of the weld zone affects the hardness as well as the metallurgical structure (Majumder, 2011).

2.4 Modelling the heat partition ratio

Compared to other metal removal processes, grinding is a high specific energy process in which almost all the grinding energy is converted into heat and is concentrated within a narrow grinding zone and this leads to several types of thermal damages to the work piece. In grinding, the wheel, work piece, coolant and chips are identified as the heat sinks. Partition ratio is defined as the proportion of total grinding energy which is partitioned to the work piece. Mathematical models of the grinding process describing the thermal phenomenon are useful for this analysis (Lavine, 1988; Guo and Malkin, 2000; Jin et al., 2003).

A step further with calorimetric approach mentioned earlier, resulted in the wheel-zone model for partition ratio (Shaw, 1990). Herein, the work piece and wheel are the heat sinks. The approach is based on the prelude that the temperature at the point of contact of wheel and work piece is the same. Hence, an expression for temperature on the wheel surface due to a heat source moving at wheel velocity can be developed and matched with T salope in Eq.(11). According to the wheel-zone model, the partition ratio is

\[
\frac{1}{\varepsilon} = 1 + \frac{(\gamma \rho c)_w V_w}{(\gamma \rho c)_w V_w}
\]  

(15)

This model is further adapted with the composite thermal properties of the wheel. The effectiveness of a coolant in the grinding zone (in wet grinding) is found to be influenced by the wheel porosity, flow rate and the

nozzle position (Malkin, 1989). The composite wheel-fluid thermal property was estimated and found useful in the partition ratio calculations in creep feed grinding (Kim et al., 1997). Assuming that the surface porosity is completely filled with grinding fluid and that the thermal properties of the composite can be approximated by the weighted volumetric average thermal properties of the grain and grinding fluid (Guo and Malkin, 1995a), the composite thermal conductivity is,

\[
k_c = \varphi_g k_f + (1 - \varphi_g) k_g
\]

(16)

and the volumetric specific heat is,

\[
\langle \rho c \rangle = \varphi_g \langle \rho c \rangle_f + (1 - \varphi_g) \langle \rho c \rangle_g
\]

(17)

Therefore, considering the workpiece, wheel-fluid composite, coolant and chips as the possible heat sinks, the expression for heat partition at the grinding zone can be written as

\[
\frac{1}{\varepsilon} = 1 + \left(\frac{(\gamma \rho c)_w V_w}{(\gamma \rho c)_w V_w} \frac{u - u_{ch}}{u}\right)
\]

(18)

By another perspective, the partition ratio can be evaluated from the grain-zone model due to R. Hahn (Rowe et al., 1995; Zarudi and Zhang, 2002; Marinescu et al., 2004), considering that the total grinding energy is partitioned between the work piece and abrasive grain. This model does not account for the heat going to chips. The partition ratio by grain-zone model is,

\[
\frac{1}{\varepsilon} = 1 + \frac{k_g}{(r_c V_c)^{1/2} (\gamma \rho c)_0^{1/2}}
\]

(19)

3. TEMPERATURE PROFILE IN THE WORK PIECE

Analysis of the thermal profile is expected to give a better understanding of thermal damages and to be able to model the heat partition ratio. The temperature profile is seen to be dependent on wheel-work contact length, thermal properties of the wheel and work piece, and the nature of heat input and cooling, as per the Jaeger’s model in Eq.(8). But, the temperature dependent variation of specific heat capacity and thermal conductivity of the work material were found to offset each other and hence negligible (Isenberg and Malkin, 1975). Moreover, there is a probability of coolant to be kept away from the grinding region by a barrier of moving air around the wheel (Malkin, 1989; Gangopadhyay et al., 1998), hence dry grinding condition was assumed in the prediction of thermal profile.

The computer program developed (Section 2.1) was utilized to estimate the temperature distribution and rate of temperature rise in work piece for a selected heat source and a set of grinding conditions. Typical grinding
conditions are given in Table 1, with specific changes noted in the corresponding figure.

The program uses an empirical relation between the grinding parameters (represented as equivalent chip thickness) and the grinding energy (Mishra and Prasad, 1985). The computer simulation approach merits simplicity against the computationally intensive finite element approach.

Table 1 Grinding Conditions and Typical Material Properties used in Analysis

<table>
<thead>
<tr>
<th>Grinding Conditions &amp; wheel-work properties</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Work material</td>
<td>Steel</td>
</tr>
<tr>
<td>Thermal Conductivity of work (γ)</td>
<td>40 W/m°K</td>
</tr>
<tr>
<td>Thermal Diffusivity of work (α)</td>
<td>0.00001323 m²/s</td>
</tr>
<tr>
<td>Thermal diffusivity of wheel (alumina)</td>
<td>1.9x10^{-6} m²/s</td>
</tr>
<tr>
<td>Wheel Speed (Vₛ)</td>
<td>30 m/s</td>
</tr>
<tr>
<td>Range of Work Speed (V𝑤)</td>
<td>0.08 to 0.45 m/s</td>
</tr>
<tr>
<td>Range of Depth of Cut (a)</td>
<td>10 to 18 μm</td>
</tr>
<tr>
<td>Wheel-Work Contact length (l𝑐)</td>
<td>0.003 m</td>
</tr>
<tr>
<td>Partition Ratio (ε)</td>
<td>0.85</td>
</tr>
<tr>
<td>Equivalent diameter (dₑ)</td>
<td>0.5 m</td>
</tr>
<tr>
<td>Wheel width (b)</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Range of Heat flux (q)</td>
<td>26.4 to 137.7 W/mm²</td>
</tr>
<tr>
<td>Specific grinding energy (u)</td>
<td>50 J/mm³</td>
</tr>
</tbody>
</table>

4. RESULTS AND DISCUSSION
4.1 Prediction of temperature profile
The temperature variations at the surface of the work piece, predicted under three different moving source of heat are shown in Figure 3. Under the same grinding conditions of the cited reference (Snoyes et al., 1978), thermal profile and parametric influence on temperature are plotted in Figures 3 through 7. The experimental temperature profile determined by Snoyes et al. (1978), was compared with the results of our theoretical model. Temperature distribution produced by triangular heat flux was comparable to the experimental trace, with an error of 10%, though it slightly under predicts at the trailing end.

Moreover, from Figure 3, it can be also observed that, the temperature rise is rather slow for a rectangular heat source when compared to that of a triangular heat source. The parabolic heat source predicts a peak temperature higher than the other two.

Figure 3 Temperature at the contact zone under different heat sources.

The temperature distribution along the ground surface for a triangular heat source at different depths below the ground surface is shown in Figure 4. Here, the distance along the ground surface is expressed in terms of the half-length of heat source (i.e.) the non-dimensional parameter (x/l) for easy comparison. It can be observed that, the peak temperature occurs just behind the heat source centre and it further lags behind for increasing depths below the surface.

Figure 4 Temperature distributions in the work piece at various depths below the surface.

4.2 Effect of grinding parameters on peak temperature
The effect of work speed on the temperature distribution along the surface is shown in Figure 5. It can be seen
that, at higher work speeds the width of heat affected zone is narrower, though the magnitude is higher at the centre of the heat source.

The rate of temperature rise under different types of heat sources is shown in Figure 7. The rate of change of temperature was calculated at discrete intervals along the temperature profile over time. This was based on the assumption that the reaction of a point under the heat source to drastic variations in the temperature, would be rather slow and continuous (i.e.) time-dependent.

Figure 5 Effect of work speed on temperature along the ground surface.

Figure 6 shows the effect of work speed on the depth of penetration of grinding heat. When the work speed is high, the surface temperature is also high but the corresponding depth of penetration is low. Figures 5 and 6 suggest that the heat affected zone is relatively smaller at high work speeds, though temperature is of a higher magnitude.

Figure 6 Depth of penetration of grinding temperature.

4.3 Analysis of heating rate on work surface
A closer look at the temperature distribution at high $V_w$ (Figure 5) shows that there is a change in the slope of the distribution curve, behind the heat source centre. So, for higher order parameters, the rate of temperature rise or drop differs significantly.

Figure 7 Rate of Temperature rise under different heat sources.

It is observed that after reaching a peak value at 30 ms, the temperature drops at a rate of around 15°C/ms and this quenching effect persists up to 60 ms. Subsequently temperature attains a steady value. The trend shown by the triangular heat source closely follows that of experimental data (Snoyes et al., 1978), whereas the rectangular and parabolic heat sources show an extended heating phase. This once again emphasizes that the triangular heat source is the best applicable model of the heat flux distribution in the grinding zone.

The rate of temperature rise in front of the heat source centre is of the order of 10 to 30°C/ms. Just after reaching the maximum value, the surface temperature drops at a rate of nearly 15°C/ms.

The temperature distribution beneath the ground surface and the corresponding rate of temperature rise are shown in Figure 8 and 9 respectively, for another grinding condition, viz, $V_v=24$m/s, $V_w=0.2$m/s, $a=10\mu$m, $l=0.002$m, $\varepsilon =0.85$, $q=27.2$ W/mm$^2$. The contours are plotted at different depths below the surface. Heating rate of 5 – 20°C/ms can be noted over 0.8 mm depth.

4.4 Consideration of the effect of heating rate on the phase change associated burn in grinding
The modelling of burn incidence reviewed in section 2.3 indicates that grinding burn in steel work pieces is associated with a localized phase change. But the critical temperature for phase change is elevated due to heating.
rate by the moving heat source. So the tendency to burn is retarded.

Figure 8 Temperature distribution in the work piece at various depth below the surface.

Figure 9 Rate of Temperature Rise in the work piece at different depth below the surface.

Referring the investigations on transformation kinetics and the anticipated heating rate of 5 - 20°C/ms (Figure 8) in grinding, the peak temperature for burn marks may be exceeding 800°C up to 910°C. Spark temperature measurement of 900°C at burn for steel work pieces, is found to be coincident (Nathan et al., 1999). So, ‘no burn’ grinding could be performed with the upper bound of temperature relaxed over 10-16% of $A_{c1}$ value.

4.5 Estimation of heating rate
Among other factors, the work speed, the length of contact and the work material properties, significantly influence the heating rate in grinding. A parameter, which combines the work speed, the length of contact and the work material properties, is the Peclet Number (Guo and Malkin, 1995b).

$$P_e = \frac{V_w l_c}{4 \left( \frac{\gamma}{\rho c_w} \right)}$$  \hspace{1cm} (20)

Hence, the $P_e$ Number could be related to the average heating rate as in Figure 10.

Figure 10 Variation of average rate of temperature rise with Peclet Number.

The y-axis shows the average heating rate at the leading edge of heat source. From the temperature profile data for a range of $V_w$ and $l_c$, the mean rate of temperature rise was calculated and plotted against corresponding $P_e$ number. Based on high or low values of $P_e$ number (and hence of $V_w$, $l_c$), the average heating rate is read from the graph in order to check if the peak temperature would cause phase changing burn.

4.6 Analysis of energy partition ratio
All of the thermal damages can be reduced and possibly avoided by careful choice of grinding conditions and a grinding wheel having favourable properties. A study into the wheel and work piece thermal properties in particular, that affect the heat partition ratio will directly gives an idea of thermal severity. Hence, the partition ratio was calculated using grain zone model, under typical dry grinding conditions for three different wheel types.

By combining the fraction of heat carried away by the chips $\varepsilon_1$, from Eq.(12), into the grain- zone model, the total heat available at the wheel wok interface is $\varepsilon_1 q$. A correction may be applied to $\varepsilon$ obtained from Eq.(19), so that partition ratio is $(\varepsilon_1 q)$. For grinding steel, $\varepsilon_1 \approx 0.85$ and varies with specific grinding energy (shown in Figure 11). For dry grinding with alumina, the partition ratio based on grain zone model is 0.81, with typical values of $V_w=30$m/s, $r_w=120$ µm and $(kpc)_w=1.4352\times10^8$. 

28
The partition ratio estimated by grain zone model is found to be comparable with experimental data, viz., \( \varepsilon \) ranges from 0.6 to 0.8 while grinding steel (Guo and Malkin, 1995a). The lower values of \( \varepsilon \) estimated by wheel zone model could be attributed to shortcomings of temperature matching (Zhang and Faghri, 1996). Nevertheless, due to complexity of the grinding process, the heat partition ratio is often validated by experimental approach (Kohli et al., 1995).

Considering the effective wheel porosity of a conventional vitrified alumina wheel, \( \phi_w = 40\% \) to 60\%, the composite thermal properties of the grinding wheel with water based fluids can be estimated to be \( k_{pc} = 0.464 \times 10^8 \text{ J/m}^2\text{K}s \) and for the work piece, \( k_{pc_w} = 1.4352 \times 10^8 \text{ J/m}^2\text{K}s \) (Rowe, 2001; Kim et al., 1997). Calculations based on the wheel zone model given by Eq.(18), under typical wet grinding conditions yielded a partition ratio of 0.10 to 0.28.

Partition ratio calculated by grain zone model after considering the heat carried away by chips, are shown in Table 2, for three different abrasives, with their widely quoted \( k_g \) values (Marinescu et al., 2004). Typical grinding conditions employed in the calculation are \( V_c = 30 \text{m/s}, r_w = 120 \mu\text{m}, (k_{pc})_w = 1.4352 \times 10^8 \text{ J/m}^2\text{K}s \). As observed in the Table 2, Cubic Boron Nitride (CBN) abrasive has the lowest energy partition ratio while Silicon Carbide (SiC) and Aluminium oxide abrasives have partition ratio in the increasing order. The difference in \( \varepsilon \) is attributed to their \( k_g \).

It can be directly inferred from the grain zone model that, smaller sized abrasive grains would result in lesser energy partition. Considering that the wear flat area strongly influences the order of grinding temperatures and that there is a critical wear flat area crossing which burn is expected to occur, it is interesting to note that the grain size of the wheel influences the wear process.

### Table 2 Partition ratio calculated by grain-zone model.

<table>
<thead>
<tr>
<th>Abrasive type</th>
<th>Grain thermal conductivity ( k_g ), W/m K</th>
<th>Partition ratio ( \varepsilon )</th>
<th>CCBN</th>
<th>Partition ratio ( \varepsilon_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al(_2)O(_3)</td>
<td>35</td>
<td>0.95</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>SiC</td>
<td>100</td>
<td>0.88</td>
<td>0.75</td>
<td></td>
</tr>
<tr>
<td>CBN(^{1})</td>
<td>240</td>
<td>0.75</td>
<td>0.64</td>
<td></td>
</tr>
</tbody>
</table>

\(^{1}\)Other references quote thermal conductivity value for CBN around 400 W/m K to 1300 W/m K (Rowe et al., 1995).

### 5. CONCLUSIONS

The temperature distribution in the work piece during grinding has been theoretically predicted using the Jaeger’s moving heat source model and the following conclusions are drawn.

1. Triangular heat flux distribution was found to be a better choice to represent the grinding heat flux.

2. Effect of grinding parameters on the peak temperature has been studied. The grinding energy, work speed and wheel-work contact length are the key parameter influencing the heat affected zone.

3. The high heating rates at the work surface has the effect of elevating the \( A_e \) temperature in steel and hence the temperature for burn is elevated. So grinding parameters can be chosen liberally without succumbing to burn and with the advantage of grinding by lesser force due to thermal softening of work piece.

4. The grain zone model gives a meaningful estimate of heat partition. Analysis of partition ratio revealed that proper choice of wheels (with high \( k_g \)), fine grain abrasives and the usage of a coolant are essential in reducing the thermal damages.

### REFERENCES


Nathan, R.D., Vijayaraghavan, L. and Krishnamurthy, R. 1999. In-process monitoring of grinding burn in the


