**PRANDTL NUMBER EFFECT ON FREE CONVECTIVE FLOW IN A SOLAR COLLECTOR UTILIZING NANOFLUID**

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**ABSTRACT**

Inside a prism shaped solar collector with a right triangular cross sectional area, a numerical model is presented for the simulation of double-diffusive free convection flow. This model is encountered in greenhouse solar stills where temperature and concentration gradients between the water-CuO nanofluid and transparent inclined cover plate induce flows in a limited space. In the water heating process and in the biological comfort this phenomenon plays an important function. In this problem, the ratio \( Nr \) of the relative magnitude thermal and compositional buoyancy and Prandtl number \( Pr \) are key parameters. Finite element technique is used to solve the governing equations with necessary boundary conditions. Numerical results are presented for the effect of the abovementioned parameters on temperature, mass and velocity distributions systematically. In addition, results for the average radiative, convective heat and mass transfer, mean temperature and concentration of nanofluid, mid height horizontal-vertical velocities and sub domain average velocity field are offered and discussed for the mentioned parametric conditions. Some interesting results are found in this investigation. The code validation shows excellent concurrence with the hypothetical outcome obtainable in the literature.

**Keywords**: Prandtl number; free convection; buoyancy ratio; prism shaped solar collector; finite element method.

**1.0 INTRODUCTION**

Solar energy, radiant light and heat from the sun, has been harnessed by humans since ancient times using a range of ever-evolving technologies. Solar energy has the greatest potential of all the sources of renewable energy especially when other sources in the country have depleted. Because of the desirable environmental and safety aspects it is widely believed that solar energy should be utilized instead of other alternative energy forms, even when the costs involved are slightly higher.

The fluids with solid-sized nanoparticles suspended in them are called “nanofluids”. The natural convection in enclosures continues to be a very active area of research during the past few decades. Applications of nanoparticles in thermal field are to augment warmth transport from solar collectors to luggage compartment tanks, to pick up proficiency of coolants in transformers. The flat-plate solar collector is commonly used now a day for the gathering of small warmth planetary thermal power. Solar collectors are key elements in many applications, such as building heating systems, solar drying devices etc.

Solar technologies are broadly characterized as either passive solar or active solar depending on the way they capture, convert and distribute solar energy. Solar energy technologies include solar heating, solar photovoltaic, solar thermal electricity and solar architecture, which can make considerable contributions to solving some of the most urgent problems the world now faces. Active solar techniques include the use of photovoltaic panels and solar thermal collectors to harness the energy. Passive solar techniques include orienting a building to the Sun, selecting materials with favorable thermal mass or light dispersing properties, and designing spaces that naturally circulate air. The development of affordable, inexhaustible and clean solar energy technologies will have huge longer-term benefits. It will increase countries’ energy security through reliance on an indigenous, inexhaustible and mostly import-independent resource, enhance sustainability, reduce pollution, lower the costs of mitigating climate change, and keep fossil fuel prices lower than otherwise. These advantages are global. Hence the additional costs of the incentives for early deployment should be considered learning investments; they must be wisely spent and need to be widely shared.
Conventional analysis and design of solar collector is based on a one-dimensional conduction equation formulation by Sukhatme (1991). Another category of collectors is the uncovered or unglazed solar collector of Soltan (1992). These are usually low-cost units which can offer cost effective solar thermal energy in applications such as water preheating for domestic or industrial use, heating of swimming pools of Molineaux et al. (1994), space heating and air heating for industrial or agricultural applications. The principal requirement of the solar collector is a large contact area between the absorbing surface and the air. Piao et al. (1994) investigated experimentally natural, forced and mixed convective heat transfer in a cross-corrugated channel solar air heater. An innovative idea is to suspend ultra fine solid particles in the fluid for improving the thermal conductivity of the fluid by Hetsroni and Rozenblit (1994). Detailed experimental and numerical studies on the performance of the solar air heater were made by Gao (1996). Noorshahi et al. (1996) studied numerically the natural convection effect in a corrugated enclosure with mixed boundary conditions. Stasiek (1998) made experimental studies of heat transfer and fluid flow across corrugated and undulated heat exchanger surfaces.

Various applications of solar air collectors were reported by Kolb et al. (1999). The absorptance of the collector surface for shortwave solar radiation depends on the nature and colour of the coating and on the incident angle. Usually black colour is used. Various colour coatings had been proposed by Tripanagnostopoulos et al. (2000), Wawrzaz et al. (2002) and Orel et al. (2002); mainly for aesthetic reasons. A low-cost mechanically manufactured selective solar absorber surface method had been proposed by Konttininen et al. (2003). There are so many methods introduced to increase the efficiency of the solar water heater by Xiaowu and Hua (2005), Xuesheng et al. (2005), Ho and Chen (2006) and Hussain (2006). But the novel approach is to introduce the nanofluids in solar water heater instead of conventional heat transfer fluids (like water). The poor heat transfer properties of these conventional fluids compared to most solids are the primary obstacle to the high compactness and effectiveness of the system. The essential initiative is to seek the solid particles having thermal conductivity of several hundred times higher than those of conventional fluids. These early studies, however, used suspensions of millimeter or micrometer-sized particles, which, although showed some enhancement, experienced problems such as poor suspension stability and hence channel clogging, which are particularly serious for systems using mini sized and micro sized particles. The suspended metallic or nonmetallic nanoparticles change the transport properties and heat transfer characteristics of the base fluid. Stability and thermal conductivity characteristics of nanofluids were performed by Hwang et al. (2007). In this study, they concluded that the thermal conductivity of ethylene glycol was increased by 30%.

A numerical experiment is performed for inclined solar collectors by Varol and Oztop (2007). Kent (2009) studied laminar natural convection in isosceles triangular enclosures for cold base and hot inclined walls numerically. Bég et al. (2011) performed non-similar mixed convection heat and species transfer along an inclined solar energy collector surface with cross diffusion effects, where the resulting governing equations were transformed and then solved numerically using the local non-similarity method and Runge-Kutta shooting quadrature. Esfahani and Bordbar (2011) studied double diffusive natural convection heat transfer enhancement in a square enclosure using nanofluids. At the same year, Nield and Kuznetsov (2011) conducted the onset of double-diffusive convection in a nanofluid layer and the Cheng–Minkowycz problem for the double-diffusive natural convective boundary layer flow in a porous medium saturated by a nanofluid. The stability boundaries for both non-oscillatory and oscillatory cases had been approximated by simple analytical expressions.

Rahman et al. (2012) performed double-diffusive natural convection in a triangular solar collector where the effects of the thermal Rayleigh number and buoyancy ratio were presented by streamlines, isothersms, iso-concentration as well as local and mean heat and mass transfer rates. Double diffusive natural convection in a partially heated enclosure using nanofluid was studied by Parvin et al. (2012) where the effect of the parameters namely Rayleigh number and solid volume fraction of nanoparticle on the flow pattern and heat and mass transfer had been depicted.

From the current study, the convective phenomenon for the effect of Prandtl number $Pr$ and buoyancy ratio...
Nr is studied through the prism shaped solar collector utilizing water-CuO nanofluid. Thus, the objective is to present flow, temperature and concentration augmentation used to harness solar energy.

2.0 GEOMETRICAL MODELING

The graphical representation of a prism shaped solar collector is expressed in the Figure 1. The liquid inside the collector is water-based nanofluid including CuO nanoparticles. The nanofluid is considered incompressible and the flow is assumed as laminar. It is taken that base fluid and nanoparticles are in thermal balance and no slip maintains among them. The solar collector is a metal box with a cover on the slant surface and a dark colored absorber plate on the bottom and vertical surfaces. The inclined wall has initially constant temperature $T_b$, while bottom and vertical walls are heated by $T_c$, with $T_b > T_c$. The concentration in inclined wall is maintained higher than other walls ($C_r < C_b$). The concentration of the nanofluid is observed by the Boussinesq form. Only steady state case is considered.

![Fig. 1 Schematic diagram of the solar collector](image)

2.1 Mathematical Formulation

The leading equations for laminar buoyancy convective flow in a solar collector utilized by water-CuO nanofluid in terms of the Navier-Stokes and energy conservation equation (dimensional form) are given as:

Continuity equation:
\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  
(1)

Momentum conservation equations:
\[
\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = - \frac{\partial p}{\partial x} + \mu_f \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]  
(2)
\[
\frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = - \frac{\partial p}{\partial y} + \mu_f \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \frac{\partial q}{\partial t} \beta_f \left( T - T_c \right) + \left( c - C_r \right)
\]  
(3)

Energy conservation equation:
\[
\frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{k}{\rho c_p} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{\partial q}{\partial t} \beta_f \left( T - T_c \right) + \left( c - C_r \right)
\]  
(4)

Concentration conservation equation:
\[
\frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = \frac{k_f}{\rho C_p} \left( \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right) + \frac{\partial q}{\partial t} \beta_f \left( T - T_c \right) + \left( c - C_r \right)
\]  
(5)

Where, $\rho_f = (1-\phi) \rho_f + \rho_p$ is the density
\[
(\rho C_p)_r = (1-\phi) (\rho C_p)_f + \phi (\rho C_p)_s
\]
the heat capacitance,
$\beta_f = (1-\phi) \beta_f + \phi \beta_s$ is the thermal expansion coefficient and
$\alpha_f = k_f / (\rho C_p)_f$ is the thermal diffusivity.

In the current study, the viscosity of the nanofluid is considered by Pak and Cho correlation (1998). This correlation is given as
\[
\mu_f = \mu_f \left( 1 + 39.11\phi + 533.9\phi^2 \right)
\]  
(6)

The effective thermal conductivity of the nanofluid is approximated by the Maxwell-Garnett model (1904):
\[
k_{E} = \frac{k_f + 2k_f - 2\phi(k_f - k_s)}{k_f + 2k_f + \phi(k_f - k_s)}
\]  
(7)

Radiation heat transfer by the inclined glass cover surface must account for thermal radiation which can be absorbed, reflected, or transmitted. This decomposition can be expressed by,
\[
q_{net} = q_{absorbed} + q_{transmitted} + q_{reflected}
\]

Outside the boundary layer, the amount of energy $q_{reflected}$ is neglected.

Thus total energy of the glass cover plate becomes
\[
q_{net} = q_{absorbed} + q_{transmitted}
\]

Now the amount of transmitted energy is radiated from the cover plate to the bottom and vertical absorber walls without any medium as:
\[
q_{transmitted} = q_{r} = \varepsilon \sigma A \left( T_w^4 - T_c^4 \right)
\]
Here $\varepsilon$ is emissivity of the glass cover plate, $\sigma$ is Stefan Boltzmann constant \(5.670400 \times 10^{-8} \text{Js}^{-1} \text{m}^{-2} \text{K}^{-4}\) and $T_w$ is the variable warmth of the slant surface. Again, the amount of absorbed energy is transferred from cover plate to absorber by natural convection where medium is nanofluid as:

$$q_{\text{absorbed}} = q_c = hA(T_w - T_c)$$

So total energy gained by the cover plate is

$$q_{\text{net}} = hA(T_w - T_c) + \varepsilon \sigma A(T_w^4 - T_c^4)$$

The boundary conditions are:

- at all concrete borders: $u = v = 0$
- at the inclined cover plate: $q = hA(T_w - T_c) + \varepsilon \sigma A(T_w^4 - T_c^4)$, $C = C_b$
- at the bottom and vertical absorber walls: $T = T_c$, $C = C_c$
- at the inclined wall: $\theta = 0$, $C = C_c$

The previous equations become dimensionless by means of the subsequent non-dimensional reliant and free variables:

$$X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{v_f}, \quad V = \frac{vL}{v_f}, \quad P = \frac{\rho L^2}{\rho_f v_f^2},$$

$$\theta = \frac{T - T_c}{T_b - T_c}, \quad C = \frac{c - C_c}{C_b - C_c}$$

Then the dimensionless governing equations are

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \quad (8)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + Pr_f \frac{v_f}{v_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \quad (9)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = Pr_f \frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + Pr_f \frac{v_f}{v_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra_f Pr \left(1 - \theta \right) \frac{\rho_f}{\rho_{nf}} \frac{\partial \theta}{\partial Y}$$

$$Ra_f Pr \left(1 - \theta \right) \frac{\rho_f}{\rho_{nf}} \frac{\partial \theta}{\partial Y} = \frac{1}{Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + D_f \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) \quad (10)$$

$$Ra_f = \frac{\rho_f}{\rho_{nf}} \frac{v_f}{v_f}$$

$$Ra_f \left(1 - \theta \right) \frac{\rho_f}{\rho_{nf}} \frac{\partial \theta}{\partial Y} = \frac{1}{Sc} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) + S_f \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (11)$$

$$U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \left( \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial Y^2} \right) + S_f \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \quad (12)$$

where $Pr = \left( \frac{v}{\alpha} \right)_f$ is the Prandtl number, $Sc = \left( \frac{v}{D_f} \right)_f$ is the Schmidt number, $Ra_f = \frac{g \beta_f L^4}{\nu_f \alpha_f} (T_b - T_c)$ is the thermal Rayleigh number, $Ra_c = \frac{g \beta_c L^4}{\nu_f \alpha_f} (C_b - C_c)$ is the solutal Rayleigh number, $N_r = \frac{Ra_f}{Ra_c}$ is the buoyancy ratio number, $D_f = \left( \frac{D}{v} \right)_f \frac{k_f}{C_f C_p(C_b - C_c)(T_b - T_c)}$ is the Dufour and $S_r = \left( \frac{D}{v} \right)_f \frac{k_f}{C_f C_p(C_b - C_c)(T_b - T_c)}$ is the Soret coefficient.

The consequent border situations get the form:

- at all concrete borders: $U = V = 0$
- at the absorber surfaces: $U = V = 0$
- at the inclined wall: $\theta = 0$, $C = C_c$

2.2 Average Nusselt Number

The average Nusselt number ($Nu$) depends on thermal conductivity, warm capacitance, stickiness, stream formation of nanofluids, volume division and fractal distributions of nanoparticles. The local convective Nusselt number of the fluid at the slant cover plate is

$$Nu_c = \frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial Y}$$

The non-dimensional form of local convective heat transfer is $\bar{Nu}_c = -\frac{k_{nf}}{k_f} \frac{\partial \theta}{\partial n}$.

With the integration of the local Nusselt number over the inclined heated surface, the mean convective heat transfer at the heated and radiated wall of the prism shaped solar collector is used by Saleh et al. (2011) as $Nu_c = \frac{1}{L_s} \int_0^{L_s} \bar{N} \text{d}N$.

The radiated heat transfer rate is expressed as $Nu_r = \frac{1}{L_s} \int_0^{L_s} q \text{d}N$ where $L_s$, $N$ are the dimensionless distance end to end and coordinate at the slant hot wall in that order.

The mean Nusselt number is $Nu = Nu_c + Nu_r$.

The average Sherwood number at the concentrated surface of the solar collector is defined as

$$Sh = \frac{1}{L_s} \int_0^{L_s} \frac{k_{nf}}{k_f} \frac{\partial C}{\partial n} \text{d}N$$

The mean bulk temperature and average sub domain velocity of the fluid inside the collector may be written as $\theta = \frac{1}{V} \int_0^V \theta dV$ and $\omega = \frac{1}{V} \int_0^V \omega dV$, where $V$ the volume of the prism is shaped solar collector.
3.0 NUMERICAL SIMULATION

The Galerkin finite element method of Taylor and Hood (1973) and Dechaumphai (1999) are used to work out the dimensionless leading equations through border settings for the considered study. The equation of mass conservation has been considered as a limit and this limitation may be used to get the pressure profile. The penalty finite element technique Basak et al. (2009) are taken to work out the Equations (8) - (12), where the pressure \( P \) is removed by a penalty constant. The mass conservation equation is mechanically satisfied by big values of this penalty constant. Then the speed components (\( U, V \)), warmth (\( \theta \)) and concentration (\( C \)) are prolonged by means of a base set. The Galerkin finite element method yields the next nonlinear residual equations. Three points Gaussian quadrature is chosen to calculate the integrals in these equations. The non-linear residual equations are solved using Newton–Raphson technique to resolve the coefficients of the series. The convergence of solutions is guessed when the comparative mistake for every variable between successive iterations is verified under the convergence measure such that \( |\psi^{n+1} - \psi^n| \leq 10^{-4} \), where \( n \) is the number of iteration and \( \Psi \) is a function of \( U, V, \theta \) and \( C \).

3.1 Mesh Generation

In the finite element technique, the interconnect creation is the system to subdivide an area into a set of sub-domains, noted finite elements, control volume, etc. The separate places are called by the arithmetical gridiron, at which the variables are to be computed. It is fundamentally a distinct depiction of the numerical area on which the study is to be resolved. The computational areas with unbalanced figures by a set of finite elements create the technique a precious realistic instrument for the key of border value problems arising in different sectors of engineering. Figure 2 displays the finite element mesh of the current physical area.

3.2 Thermo-physical Properties

The thermo-physical properties of the nanofluid are taken from Ogut (2009) and given in Table 1.

3.3 Grid Independent Test

For \( Pr = 6.2, \; Nr = 1, \; Ra_T = 10^4 \) and \( \phi = 4\% \) in a prism shaped solar collector, a widespread mesh solution process is performed to assurance grid-independent test. We inspect five special non-uniform grid schemes with the subsequent number of elements inside the declaration sector: 2969, 5130, 6916, 9057 and 11426 in the present work. For the above mentioned elements to build up an accepting of the grid excellence the mathematical system is brought for extremely accurate type in the mean convective and radiative Nusselt numbers namely \( Nu_c \) and \( Nu_r \) and average Sherwood number \( Sh \). It is exposed in Table 2 and Figure 3. The level of the mean Nusselt number (convective and radiative) and Sherwood number for 9057 elements exposes a small variation with the outcome attained for the other elements. Hence, allowing for the non-uniform grid scheme of 9057 elements is chosen for the calculation.

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**Table 1** Thermo physical properties of fluid and nanoparticles

<table>
<thead>
<tr>
<th>Physical Properties</th>
<th>Fluid phase (Water)</th>
<th>CuO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p ) (J/kgK)</td>
<td>4179</td>
<td>535.6</td>
</tr>
<tr>
<td>( \rho ) (kg/m(^3))</td>
<td>997.1</td>
<td>6500</td>
</tr>
<tr>
<td>( k ) (W/mK)</td>
<td>0.613</td>
<td>20</td>
</tr>
<tr>
<td>( \alpha \times 10^7 ) (m(^2)/s)</td>
<td>1.47</td>
<td>57.45</td>
</tr>
<tr>
<td>( \beta \times 10^5 ) (1/K)</td>
<td>21</td>
<td>5.1</td>
</tr>
</tbody>
</table>

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**Table 2** Grid Sensitivity Check at \( Pr = 6.2, \phi = 4\%, \; Nr = 1 \) and \( Ra_T = 10^4 \)

<table>
<thead>
<tr>
<th>Nodes (elements)</th>
<th>6224 (2969)</th>
<th>10982 (5130)</th>
<th>13538 (6916)</th>
<th>20295 (9057)</th>
<th>27524 (11426)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Nu_c )</td>
<td>6.83</td>
<td>7.98</td>
<td>8.78</td>
<td>9.09</td>
<td>9.09</td>
</tr>
<tr>
<td>( Nu_r )</td>
<td>4.33</td>
<td>5.38</td>
<td>5.99</td>
<td>6.45</td>
<td>6.46</td>
</tr>
<tr>
<td>( Sh )</td>
<td>5.09</td>
<td>6.59</td>
<td>7.59</td>
<td>8.09</td>
<td>8.19</td>
</tr>
<tr>
<td>Time (s)</td>
<td>226.3</td>
<td>292.6</td>
<td>388.2</td>
<td>421.3</td>
<td>627.4</td>
</tr>
</tbody>
</table>
3.4 Code Validation

The present numerical resolution is authenticated by matching the current code outcome for velocity and temperature profiles with the graphical representation of Joudi et al. (2004) which was reported for calculation form for a prism shaped solar collector. Figure 4 demonstrates the above stated comparison. As shown in Figure 4, the numerical solutions and Joudi et al. (2004) are in good agreement.

In addition, the present code results for average Nusselt (Nu) and Sherwood (Sh) numbers with the variation of thermal Rayleigh number (Ra_T) using R = 0.5, N = 1, D_f = S_r = 0.5, Sc = 5 and Pr = 11.573 with the graphical demonstration of Nithyadevi and Yang (2009). The above declared comparison is demonstrated in the Figure 5.

4.0 RESULTS AND DISCUSSIONS

For different Prandtl number (Pr) and buoyancy ratio (Nr), numerical investigations of velocity, temperature and concentration with water-CuO nanofluid through a prism shaped solar collector are displayed. The solid volume fraction $\phi$ = 4%, Dufour coefficient $D_f$ = 0.5, Soret coefficient $S_r$ = 1, thermal Rayleigh number $Ra_T = 10^4$ and the emissivity $\varepsilon = 0.9$ are kept fixed. The considered values of Pr and Nr are Pr = (6.2, 7.56, 10.26 and 12.99), Nr (= 1, 5, 10 and 15) Also, for the above mentioned parameters, the mean Nusselt number both for convective and radiative as well as mean bulk temperature, concentration, average sub domain velocity profile, mid-height horizontal and vertical velocity components inside the prism shaped solar collector are revealed graphically.

The effects of Pr on the thermal, concentration and flow fields are presented in Figure 6 (a)-(c) while Nr = 1. The strength of the flow circulation, the thermal current and concentration activities are much more activated with escalating Pr from 6.2 to 12.99. Isotherms and iso-concentrations are almost similar to the active parts. Increasing Pr, the temperature and concentration lines at the middle part of the prism shaped solar collector become bended whereas initially (Pr = 6.2) they are almost parallel due to convection is dominated across the collector. With rising values of Pr, the temperature and concentration distributions become distorted resulting in an increase in the overall heat and mass transfer. This result can be attributed to the dominance of the viscous force. It is worth noting that as the Prandtl number increases, the thickness of the thermal boundary layer near the inclined surface.
enhances which indicates steep temperature and concentration gradients and hence, an increase in the overall heat and mass transfer within the solar collector. From the Figure 6(c), it is observed that the fluid rises along the left wall and falls along the slant wall in the velocity profile. A primary anticlockwise recirculation occupying the whole cavity is found at relatively lower values of the Prandtl number (Pr). The size of the eye of this cell becomes larger upto Pr = 10.26. With the highest value of the Prandtl number (Pr = 12.99), this cell becomes smaller in size. In addition two small eddies are created both in top and right corner of the solar collector. This happens due to increasing viscous force. Highly viscous nanofluid can’t move freely.

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For the escalating values of buoyancy ratio Nr, the thermal current activities are much more activated and thermal boundary layer near the top inclined surface enhances. On the other hand, Figure 7 (b) describes that iso-concentration lines tend to move near radiated and inclined surface due to rising Nr. As a result, empty space is created in the vicinity of left bottom corner of the solar collector and first two concentration lines changes their pattern from triangular to circular. This leads to formation of the solutal boundary layers at the right base part and peak place of the absorber walls. These mean that higher heat and mass transfer rates are predicted by the water-CuO nanofluid for higher values of buoyancy ratio. From the Figure 7(c) we observe that at the lowest value of Nr = 1 the fluid flow covers the entire collector in a formation of a primary cell. This cell rotates clockwise direction. The shape of this major cell is triangular. Due to rising Nr the core of this primary vortex becomes larger at the active part of the collector. This is expected because nanofluid that is fluid with solid concentration moves rapidly with the effect of mounting buoyancy ratio.

Figure 7 (a)-(c) depicts the results of heat, mass transfer and fluid flow for various buoyancy ratio Nr with Pr = 6.2. As the buoyancy ratio enhances from 1 to 15, the isotherms and iso-concentrations contours tend to get affected considerably. It is seen from Figure 7 (a) that the temperature lines corresponding to Nr = 15 become more bended. But, they are almost flatten near the active location on the inclined transparent surface of the prism shaped solar collector for Nr = 1.

Fig. 6 Effect of Pr on (a) isotherms, (b) iso-concentration and (c) streamlines at Nr = 1

Fig. 7 Effect of Nr on (a) isotherms, (b) iso-concentration and (c) streamlines at Pr = 6.2
The average Nusselt (convective and radiative) numbers and mean Sherwood number (Sh) profile along with the various Prandtl number (Pr) and buoyancy ratio (Nr) are displayed in Figure 8 (i)-(ii). From this figure it is evident that a linear increasing is obtained according to the buoyancy ratio. It is seen from Figure 8(i) that $N_u_c$, $N_u_r$ and Sh enhance with the growing values of Prandtl number. The reason is that rising Pr causes lower temperature of the working fluid. From Figure 8(ii) it is evident that a linear increasing is obtained among convective heat and mass transfer rates according to the buoyancy ratio. Mean radiative Nusselt number remains almost invariant. Rate of convective, radiative heat and mass transfer enhances by 16%, 8% and 12% respectively with the variation of Pr from 6.2 to 12.99. Rate of convective and radiative heat transfer enhances by 6% and 2% whereas this rate for mass transfer is 5% with the increasing values of Nr from 1 to 15. It is well known that rate of heat transfer is always higher for convection than radiation and is justified by the current investigation.

The average temperature ($\theta_{av}$) and concentration ($C_{av}$) along with the Prandtl number (Pr) and buoyancy ratio (Nr) are depicted in Figure 9(i)-(ii). They reduce with escalating Pr and Nr. This happens because temperature of fluid devalues for the higher values of Prandtl number and buoyancy ratio.

Figure 10 (i)-(ii) displays the mid height horizontal (U) velocity component for the effect of Pr and Nr inside the prism shaped solar collector. It is observed that the fluid particle moves with greater velocity for the lowest value of Pr (viscosity parameter). The waviness in the velocity profile diminishes for higher values of Pr. On the other hand, in the U velocity profile at the middle of the collector a considerable variation is found due to changing Nr. It is desired that, working fluid moves with greater velocity due to the influence of buoyancy ratio.

The mid-height vertical velocity profile for different Pr and Nr effects are expressed in Figure 11 (i)-(ii). With the highest Pr and lowest Nr fluid particle moves with lower velocity. This is happened due to the dominating viscous force and buoyancy ratio of the water-CuO nanofluid.

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**Fig. 8** Plots of $N_u_c$, $N_u_r$ and Sh with various (i) Pr and (ii) Nr

**Fig. 9** Plots of $\theta_{av}$ and $C_{av}$ with various (i) Pr and (ii) Nr

**Fig. 10** Plots of mid height horizontal velocity with various (i) Pr and (ii) Nr

**Fig. 11** Plots of mid height vertical velocity with various (i) Pr and (ii) Nr
Figure 12 (i)-(ii) displays the average sub domain velocity field $\omega_{av}$ inside the prism shaped solar collector for the influence of Prandtl number and buoyancy ratio. From Figure 12 (i), it is found that $\omega_{av}$ grows down and up with the variation of Pr and Nr respectively.

5.0 CONCLUSION

Inside a prism shaped solar collector with water-CuO nano fluid, the behavior of nanoparticles on free convective boundary layer flow is accounted. For the observation of flow, temperature and concentration fields as well as the heat (convective and radiative) and mass transfer rate, horizontal and vertical velocities at the middle height of the collector various Prandtl number and buoyancy ratio have been considered. In the current study $\phi$, $D_f$, $\epsilon$, $Ra_T$ and $S_r$ are considered as 4%, 0.5, 0.9, $10^4$ and 1 respectively. Following conclusions have been drawn from the results of the numerical analysis:

- The configuration of the fluid streamlines, isotherms and iso-concentrations within the solar collector is found to significantly depend upon both the parameters namely Pr and Nr.
- The CuO nanoparticle with the highest Pr and Nr is established to be most effective in enhancing performance of heat and mass transfer rates.
- Greater variation is observed in (U and V) velocity components at a particular point for the falling Pr and growing buoyancy ratio.
- Mean temperature enhances by 28% and 26% with decreasing values of Pr and Nr respectively.
- Rising Pr and Nr result lessening average concentration by 26% and 22% respectively.
- Average velocity field increases 31% and 33% due to diminishing Prandtl number and mounting buoyancy ratio respectively.

**NOMENCLATURES**

- **A** Area of glass cover plate (m$^2$)
- **c** Dimensional concentration (kg m$^{-3}$)
- **C** Non-dimensional concentration
- **$C_p$** Specific heat at constant pressure (J kg$^{-1}$ K$^{-1}$)
- **$C_s$** Concentration susceptibility
- **$D_s$** Solutal diffusivity (m$^2$ s$^{-1}$)
- **$D_f$** Dufour parameter, $D_f = \left(\frac{D}{v_f}\right) \frac{k_{eff} (C_h - C_e)}{C_s C_p (T_h - T_e)}$
- **g** Gravitational acceleration (m s$^{-2}$)
- **h** Local heat transfer coefficient (W m$^{-2}$ K$^{-1}$)
- **k** Thermal conductivity (W m$^{-1}$ K$^{-1}$)
- **L** Length of the solar collector (m)
- **$N_r$** Buoyancy ratio, $N_r = \frac{Ra_a}{Ra_T}$
- **Nu** Nusselt number, $Nu = hL/k$
- **Pr** Prandtl number, $Pr = \frac{v_f}{\alpha_f}$
- **$Ra_a$** Solutal Rayleigh number, $Ra_a = \frac{g \beta_{eff} L^3 (T_h - T_e)}{v_f \alpha_f}$
- **$Ra_T$** Thermal Rayleigh number, $Ra_T = \frac{g \beta_{eff} L^3 (T_h - T_e)}{v_f \alpha_f}$
- **Sc** Schmidt number, $Sc = \left(\frac{v}{D}\right)_f$
- **Sh** Sherwood number, $Sh = hL/k_f$
- **$S_r$** Soret parameter, $S_r = \left(\frac{D}{v_f}\right)_f \frac{k_{eff} (T_h - T_e)}{T_m (C_h - C_e)}$
- **T** Dimensional temperature (°K)
- **$T_i$** Initial temperature of nanofluid (°K)
- **u, v** Dimensional x and y components of velocity (m s$^{-1}$)
U, V Non dimensional velocities,
\[ U = \frac{uL}{v_f}, \quad V = \frac{vL}{v_f} \]

X, Y Non dimensional coordinates,
\[ X = x/L, \quad Y = y/L \]

x, y Dimensional coordinates (m)

Greek Symbols
\[ \alpha \] Fluid thermal diffusivity \( (m^2 \text{s}^{-1}) \)
\[ \beta \] Thermal expansion coefficient \( (K^{-1}) \)
\[ \varepsilon \] Emissivity
\[ \varphi \] Nanoparticles volume fraction
\[ \nu \] Kinematic viscosity \( (m^2 \text{s}^{-1}) \)
\[ \theta \] Dimensionless temperature,
\[ \theta = (T - T_c)/(T_h - T_c) \]
\[ \rho \] Density \( (kg \text{ m}^{-3}) \)
\[ \mu \] Dynamic viscosity \( (N \text{s m}^{-2}) \)
\[ \sigma \] Stefan Boltzmann constant
\[ \omega \] Dimensionless velocity field

Subscripts
\[ av \] mean
\[ c \] cold
\[ f \] fluid
\[ h \] hot
\[ nf \] nanofluid
\[ s \] solid particle
\[ w \] slant surface

REFERENCES


