HEAT TRANSFER ENHANCEMENT BY NANOFUID IN A CAVITY CONTAINING A HEATED OBSTACLE

S. Parvin*, K.F.U. Ahmed, M.A. Alim and N.F. Hossain
Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka, Bangladesh
*Corresponding author’s email: salpar@math.buet.ac.bd

Received 25 January 2012, Accepted 01 May 2012

ABSTRACT
This work investigates the performance of nanofluid as a heat transfer medium on natural convection around a circular heater placed in a square cavity. Water based nanofluid with Cu nanoparticle is used as the medium. Graphical representation of streamlines, isotherms and rate of heat transfer in terms of average Nusselt number for different values of Rayleigh number (Ra) and solid particle volume fraction (\( \phi \)) is shown. The outcomes obtained from finite element method reveal the considerable dependency of above mentioned parameters on the flow and thermal characteristics.

Keywords: Heat transfer enhancement, nanofluid, free convection, square cavity, heat source.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_p )</td>
<td>specific heat at constant pressure (kJkg(^{-1})K(^{-1}))</td>
</tr>
<tr>
<td>( g )</td>
<td>gravitational acceleration (ms(^{-2}))</td>
</tr>
<tr>
<td>( k )</td>
<td>thermal conductivity (Wm(^{-1})K(^{-1}))</td>
</tr>
<tr>
<td>( L )</td>
<td>length of the enclosure (m)</td>
</tr>
<tr>
<td>( Nu )</td>
<td>Nusselt number</td>
</tr>
<tr>
<td>( p )</td>
<td>dimensional pressure (Nm(^{-2}))</td>
</tr>
<tr>
<td>( P )</td>
<td>non-dimensional pressure</td>
</tr>
<tr>
<td>( Pr )</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>( Ra )</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>( T )</td>
<td>dimensional temperature (K)</td>
</tr>
<tr>
<td>( u, v )</td>
<td>velocity components (ms(^{-1})) along ( x, y ) direction respectively</td>
</tr>
<tr>
<td>( U, V )</td>
<td>dimensionless velocity components along ( X, Y ) direction respectively</td>
</tr>
<tr>
<td>( x, y )</td>
<td>Cartesian coordinates (m)</td>
</tr>
<tr>
<td>( X, Y )</td>
<td>non-dimensional Cartesian coordinates</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>thermal diffusivity (m(^2)s(^{-1}))</td>
</tr>
<tr>
<td>( \beta )</td>
<td>thermal expansion coefficient (K(^{-1}))</td>
</tr>
</tbody>
</table>

\( \phi \) solid volume fraction
\( \theta \) non-dimensional temperature
\( \mu \) dynamic viscosity of the fluid (kg m\(^{-1}\)s\(^{-1}\))
\( \nu \) kinematic viscosity of the fluid (m\(^2\)s\(^{-1}\))
\( \rho \) density of the fluid (kg m\(^{-3}\))

Subscripts

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c )</td>
<td>less heated wall</td>
</tr>
<tr>
<td>( f )</td>
<td>base fluid</td>
</tr>
<tr>
<td>( h )</td>
<td>heated wall</td>
</tr>
<tr>
<td>( nf )</td>
<td>water-Cu nanofluid</td>
</tr>
<tr>
<td>( s )</td>
<td>solid particle</td>
</tr>
</tbody>
</table>

1. INTRODUCTION
Nanofluids containing nanometer-sized particles of metals, oxides, carbides, nitrides, or nanotubes etc. play an important role to the researcher because of their superior thermal characteristics over the base fluids. For this reason nanofluid is very attractive as heat transfer fluids in many industrial applications as coolants in the automobile and electronics industries. The augmented thermal conductivity of nanofluids is accredited to various incidents counting Brownian motion, collecting of nanoparticles, and liquid layering at the liquid/solid interface. Many researchers described these issues in their works as detailed in the following section.

A model regarding heat transfer enhancement inside a two-dimensional enclosure using nanofluids was firstly investigated by Kanafer et al. (2003). The nanofluid was considered as single phase fluid. Their findings were that increase in both the nanoparticles volume fraction and Grashof number enhanced the rate of heat transfer. Jou and Tzeng (2006), Oztop and Abu-Nada (2008), Ogut (2009), Das and Ohal (2009) carried out the similar types of computations. Kumar et al. (2009) performed numerical study of nanofluid on flow and thermal behaviour by single phase thermal dispersion model. They used finite volume method. Conflicting outcomes were described by Putra et al. (2003) based on experimentation analysis of water–Cu or water–Al\(_2\)O\(_3\) nanofluids in a cylindrical cavity. They reported a contradictory behaviour of heat transfer declination and
its influence on solid particle volumes, particle materials and the enclosure geometry. Santra et al. (2008a) simulated the performance of heat transport caused by free convection in a cavity with different thermal boundary condition taking into account the nanofluid as a heat transfer fluid. They observed heat transfer reduction due to the raise in solid particle concentration and inverse of Rayleigh number.

The investigation of free convective cooling by fluids containing nanoparticles in square or rectangular cavities is the subject matter of most of the published papers. These investigations could be found in Khanfar et al. (2003), Jou and Tzeng (2006), Ogut (2009), Das and Ohal (2009), Kumar et al. (2009), Anand and Arora (2005), Rouboa et al. (2008), Santra et al. (2008b), Wong et al. (2008), Abu-Nada and Oztop (2009), Ghasemi and Aminossadati (2009), Muthamilvelyan et al. (2010) and Abu-Nada et al. (2010). In actual fact free convection phenomena in a cavity with different heat source is a trial product of numerous industrial uses. Bhattacharya et al. (2009) made a numerical analysis on laminar forced convection flow of Al2O3/H2O nanofluid inside a silicon microchannel heat sink by using thermal dispersion model. Very recently cooling performance of SiO2–water nanofluids in a miniature heat sink were examined both experimentally and numerically by Fazeli et al. (2012). Their findings exposed that size of the channel and heat sink and quantity of channels in a heat sink significantly affected the greatest temperature of the sink. Raja et al. (2012) studied experimentally the heat transfer of alumina/water nanofluid in a shell and tube heat exchanger with wire coil insert. They found significant augmentation in the overall heat transfer characteristics with the increment in volume concentration of the nanoparticles and a particular peclet number. Using finite element method Al-Rashed and Badruddin (2012) analyzed the movement of fluid and thermal energy owing to uneven heating of a porous medium confined in a cavity.

Varol et al. (2009a, 2009b) studied the free convection phenomena within a right-angle trapezoidal cavity having differentially heated walls for porous medium. They used finite difference method with usual rectangular grid and employing staircase-like zigzag shapes for the tilted walls. A mathematical analysis used to simulate laminar combined convective flow inside a sliding inclined cavity has been performed by Hussain (2010). The heat transfer and fluid flow behavior in a cavity with differentially heated side walls was reported by Azwadi and Idris (2010) using two different approaches namely finite difference formulation and lattice Boltzmann method. Billah et al. (2011) investigated MHD combined convective thermal current within a double-lid driven enclosure containing a heat-generating solid block. They showed strong dependency of block diameter, location of the block as well as mixed convection parameter on velocity plus thermal field. Mamun et al. (2010) investigated mixed convection heat transfer in a bottom heated trapezoidal cavity with a moving upper wall. They showed the consequence of Richardson number, aspect ratio, and rotation angle of the optimum trapezoidal cavity.

Moreover, Lin and Viol (2010) performed an analysis on free convective heat transfer performance of water - Al2O3 nanofluid within an enclosure bounded by differentially heated walls. They focused mainly on the influences of volume of nanoparticles on transportation of heat. In recent times, free convective heat transport of fluid contained nanoparticle suspension in a trapezoid shaped cavity had been examined by Saleh et al. (2011). They reported that cooling performance was affected by inclined wall as well as greater volume fraction of solid Cu nanoparticles. They too developed a relationship between the rate of heat transfer and the slope of the inclination, thermal conductivity and viscosity and natural convection parameter. Very recently, Nabavitabatabayi et al. (2011) investigated the heat transport augmentation by nanofluids in a cavity by multiple relaxation time lattice Boltzmann modelling.

In spite of a quantity of computational analysis on cavities was described, there is still a severe need of information concerning the dilemma of heat transfer improvement in cavities including nanofluids having heat source. In addition, there is a vital need to understand the greatest preparation for the locally heated area with the purpose of obtains the maximum thermal reaction due to the arrangement. The current analysis is an effort to incorporate this issue. Hence, the present computational study of free convection flow of water based nanofluid containing Cu nanoparticle in a square cavity including a central heat source is made trough the present paper. The streamlines and isothermal lines have been shown graphically for various combination of governing parameters namely Ra and φ. The average and normalized Nusselt number Nu and Nu*, horizontal and vertical velocity at a particular point are also presented for the mentioned parameters. The outcomes of this paper may become a helpful point for the engineers and researchers to understand the efficiency of the free convective heat transfer by nanofluids inside a cavity containing a heat source.

2. PHYSICAL MODEL

![Figure 1 Physical model](image-url)
Figure 1 shows the considered geometry of the square enclosure of length L with a heated circular body of temperature \( T_h \) placed at the centre. The vertical walls have constant temperature \( T_v \) where \( T_v < T_h \). The horizontal walls of the cavity are well insulated. The water-Cu nanofluid fluid is considered as a heat transfer medium in the cavity.

3. MATHEMATICAL FORMULATION

Steady, laminar, incompressible free convective behaviour of fluid flow is accounted in the current investigation where the viscous dissipation is neglected. The gravitational force works in the opposite of y-direction. Constant physical and thermal characteristics of the fluid are used. The Boussinesq approximation is valid. The flow is governed by the following equations of conservation of mass, momentum and energy

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

(1)

\[
\rho_f \left( \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu_f \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

(2)

\[
\rho_f \left( \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu_f \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + g \rho_f \beta_f (T - T_c)
\]

(3)

\[
u \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \alpha_f \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]

(4)

where, \( \rho_f = (1-\phi) \rho_f + \phi \rho_s \) is the density, \( \rho C_p \), \( \beta_f \), \( \mu_f \) are the heat capacitance, thermal diffusivity and dynamic viscosity and \( k_f \) is the thermal conductivity of the nanofluid.

The physical conditions applied to the boundaries are

- at the circular body surface \( \theta = 1 \)
- at the vertical walls \( \theta = 0 \)
- at the top and bottom walls \( \frac{\partial \theta}{\partial y} = 0 \)
- at all rigid boundaries \( u = v = 0 \)

The following dimensionless dependent and independent variables are used to make the above equations non-dimensional:

\[
X = \frac{x}{L}, \quad Y = \frac{y}{L}, \quad U = \frac{uL}{\alpha_f}, \quad V = \frac{vL}{\alpha_f}, \quad P = \frac{pL^2}{\rho_f \alpha_f^2}, \quad \theta = \frac{T - T_c}{T_h - T_c}
\]

After substituting the above variables in Eqs. (1) to (4), the obtained dimensionless equations are as follows:

\[
\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0
\]

(5)

\[
U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial X} + \frac{\nu_f}{\nu_f} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right)
\]

(6)

\[
U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\rho_f}{\rho_{nf}} \frac{\partial P}{\partial Y} + \frac{\nu_f}{\nu_f} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) + Ra \Pr (1-\phi) \rho_f \beta_f \phi \beta_f + \frac{\rho_f}{\rho_{nf}} \beta_f
\]

(7)

\[
U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{\alpha_f}{\alpha_f} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right)
\]

(8)

where \( Pr = \frac{\nu_f}{\alpha_f} \) is Prandtl number and \( Ra = \frac{g \beta_f (T_h - T_c) L^3}{\nu_f \alpha_f} \) is Rayleigh number.

Subsequently the non dimensional boundary conditions get the forms given below:

- at the circular body surface \( \theta = 1 \)
- at the vertical walls \( \theta = 0 \)
- at the top and bottom walls \( \theta_y = 0 \)
- at all rigid boundaries \( U = V = 0 \)

The mean Nusselt number at the boundary of the heater can be written as

\[
Nu = \frac{1}{S} \int_0^S \left( \frac{k_f}{k_f} \right) \frac{\partial \theta}{\partial N} \, dS
\]

where \( \frac{\partial \theta}{\partial N} = \sqrt{\left( \frac{\partial \theta}{\partial X} \right)^2 + \left( \frac{\partial \theta}{\partial Y} \right)^2} \) and \( S, N \) are the dimensionless periphery and normal coordinate along the heat source respectively.

A normalized mean Nusselt number given by the relation

\[
Nu^*(\phi) = \frac{Nu(\phi)}{Nu(\phi = 0)}
\]

is used to compare the average Nusselt number at any nanoparticles volume fraction to the zero volume fraction that is base fluid.
4. NUMERICAL TECHNIQUES

Finite element method of Galerkin weighted residual approach explained by Taylor and Hood (1973) and Dechaumphai (1999) is used solve the dimensionless equations of continuity, momentum and energy along with the boundary conditions. At first the solution area is divided to a collection of triangle shaped finite elements having six nodes. After that dependent variables within each element are approximated by applying interpolation functions. Then, the arrangement of the controlling equations with appropriate conditions to boundaries yields a residue for every equation. By Galerkin method these residues are then made to zero. The integration concerned in every term of these equations is carried out by Gauss’s quadrature scheme. The highly coupled nonlinear equations so acquired are customized by implementation of boundary conditions and converted to linear algebraic equations applying Newton’s technique. At last, solutions of these linear equations are obtained through Triangular Factorization method.

Figure 2 Mesh Generation

4.1 Grid Generation

Grid or mesh generation is the partition of the geometry model into small units of simple shapes named finite elements, control volume etc that approximates the physical domain in finite element method. Dependent variables are approximated at the local element coordinates defined by the numerical grid. It is mainly a disconnected demonstration of the physical domain where the solutions are to be carried out. Meshing the complicated geometry make the finite element method a powerful technique to solve boundary value problems occurring in a range of engineering applications. Figure 2 shows the mesh configuration of present physical domain with triangular finite elements.

4.2 Grid Refinement Test

Five different mesh sizes as shown in Table 1 are applied for grid independence test with $Pr = 6.2$, $Ra = 10^5$, and $\phi = 5\%$ to find out the appropriate grid size for the present study. It is observed that the value of $Nu$ has a slight difference between the mesh four (40215 nodes and 10872 elements) and five (80242 nodes and 18080 elements). Considering $Nu$ as the monitoring variable, mesh four is used to perform the overall computation for accuracy of results and saving computational time.

Table 1 Grid independence test at $Pr = 6.2$, $Ra = 10^5$ and $\phi = 5\%$

<table>
<thead>
<tr>
<th>Nodes (elements)</th>
<th>Nu</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7164 (4812)</td>
<td>8.536724</td>
<td>226.265</td>
</tr>
<tr>
<td>12298 (5746)</td>
<td>8.938752</td>
<td>292.594</td>
</tr>
<tr>
<td>26358 (8984)</td>
<td>9.929735</td>
<td>388.157</td>
</tr>
<tr>
<td>40215 (10872)</td>
<td>10.532235</td>
<td>421.328</td>
</tr>
<tr>
<td>80242 (18080)</td>
<td>10.532324</td>
<td>627.375</td>
</tr>
</tbody>
</table>

4.3 Thermo-physical characteristics

The thermo-physical characteristics of the water-Cu nanofluid are found from Saleh et al. (2011) and shown in Table 2. The value of Prandtl number is considered 6.2 for water at 300K.

Table 2 Thermo-physical characteristics of water-Cu nanofluid at $Pr = 6.2$

<table>
<thead>
<tr>
<th>Physical properties</th>
<th>water</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_p$</td>
<td>4179</td>
<td>383</td>
</tr>
<tr>
<td>$\rho$</td>
<td>997.1</td>
<td>8954</td>
</tr>
<tr>
<td>$k$</td>
<td>0.6</td>
<td>400</td>
</tr>
<tr>
<td>$\beta$</td>
<td>$21 \times 10^5$</td>
<td>$1.67 \times 10^6$</td>
</tr>
</tbody>
</table>

4.4 Code validation

The present numerical outcomes are validated with that of the study made by Nabavitabatabayi et al. (2011) in a cavity containing the local heat source. Excellent agreement is found. Table 3 shows the obtained results in terms of the rate of heat transfer with three different Rayleigh numbers at a fixed length of the heater. This validation boosts the confidence to carry on the current numerical investigation.

Table 3 Mean Nusselt number with various $Ra$ while dimensionless heat source length $E = 0.2$

<table>
<thead>
<tr>
<th>$Ra = 10^5$</th>
<th>$Ra = 10^4$</th>
<th>$Ra = 10^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Present work</td>
<td>5.9141</td>
<td>5.9486</td>
</tr>
<tr>
<td>Nabavitabatabayi et al. (2011)</td>
<td>5.904</td>
<td>5.952</td>
</tr>
</tbody>
</table>

5. RESULTS AND DISCUSSION

In the present investigation, the impacts of controlling parameters namely Rayleigh number $Ra$ and solid particle volume fraction $\phi$ on the streamlines and isotherms are performed. While Prandtl number is fixed at $Pr = 6.2$. The considered values of $Ra$, and $\phi$ are $Ra = (10^3, 10^4$ and $10^5)$ and $\phi = (0\%, 5\%, 10\%$ and $20\%)$. Besides these, the mean and normalized Nusselt numbers as well as horizontal and vertical velocities at a particular point in the enclosure have been calculated for different mentioned parameters.
The effects of $Ra$ upon the thermal and velocity fields have been presented in Figure 3 (a) - (b) while $Pr = 6.2$ and $\phi = 5\%$. Two primary recirculation cells are found in the streamlines at relatively lower values of the Rayleigh number ($Ra$). The shapes of these vortices change from circular to rectangular with the increasing $Ra$. This happens as a result of getting higher buoyancy force. The strength of the flow circulation and the thermal current activities are much more activated with escalating $Ra$. For increment of $Ra$, the isothermal lines become more condensed near the heat source and a thermal plume formed based on the heated body due to convection is dominated across the enclosure. Due to rising values of $Ra$, the temperature distributions become deformed ensuing in an augmentation in the on the whole heat transport process. This effect may be accredited to the control of the buoyant convection. Moreover, it is observed that raising the Raleigh number causes the higher depth of the thermal boundary layer near the heated surface that point towards a steep temperature gradient and consequently, an enhancement on the whole heat transfer inside the cavity.

The results are presented in terms of isotherms and streamline for different choice of solid volume fraction $\phi$ in Figure 4 (a) - (b). It is noted from the figures that the stream functions occupies the whole domain in the absence of $\phi$ that is, the case of base fluid. The streamlines have no significant change due to rising $\phi$ except the core of the vortex becomes slightly larger. The shape of streamlines changes slightly with the increasing of the solid volume fraction because of higher concentration of nanoparticles. As the volume fraction of nanoparticles enhances from 0% to 20%, the isotherm contours tend to get affected considerably.

In addition, these lines corresponding to $\phi = 20\%$ become more bended. The isotherms are packed out about the vigorous part of the heated surface in the cavity for clear water ($\phi = 0\%$). Rising $\phi$ shows a deformation at the isothermal lines near the upper part of the top portion of the heat source.

Figure 5 (a) - (b) displays the mean and normalized Nusselt number due to the Rayleigh number ($Ra$) as well as solid volume fraction $\phi$ effect. It is seen from figures that $Nu$ enhance sharply upto $Ra = 10^4$ and beyond this region they rise gradually. $Nu$ increases for greater values of $\phi$ because nanofluid has greater thermal conductivity in comparison to pure water. The normalized $Nu$ is similar in pattern for upper values of $\phi$ while it has no change in the absence of $\phi$ which is expected.

Figure 6 (a) - (b) demonstrates the quarter-height velocity contours at $X$ and $Y$ direction for various $Ra$. The figures indicate that the fluid elements move with larger speed at the larger values of $Ra$ number. The waviness devalues for lower values of $Ra$.

The $U$ and $V$ velocities at the quarter height of the enclosure at various $\phi$ are depicted in Figure 7 (a) - (b). No significant variation in velocity is found due to changing $\phi$. In the case of base fluid, some perturbations are seen in the velocity graph for both horizontal and vertical directions. It is desired that, clear water moves more rapidly than the solid concentrated nanofluid.
Figure 4 (a) Isotherms and (b) Streamlines for various $\phi$ with $Ra = 10^4$

Figure 5 (a) Average and (b) Normalized Nusselt numbers for various $\phi$ and $Ra$

Figure 6 (a) horizontal velocity at $(X, 0.25)$ and (b) vertical velocity at $(0.25, Y)$ for various
6. CONCLUSION
A numerical investigation concerning the effects of nanoparticle concentration and natural convection parameter $Ra$ on velocity and temperature field around a circular heat source placed in an enclosure filled with water-Cu nanofluid is accounted. The focal point of the present investigation is to calculate the average heat transfer rate, $U$ and $V$ velocities at the quarter height of the enclosure with a wide choice of Rayleigh number along with solid volume fraction while $Pr$, is fixed at 6.2. The subsequent findings can be seen from the current numerical analysis:

- $Ra$ and $\phi$ significantly affect the configuration of the streamlines and isotherms within the enclosed space.
- The Cu nanoparticle with the highest $Ra$ is established as most efficient in improving the rate of cooling. The percentage of this augmentation is found 4%-10% for different orientation of considered $\phi$.
- Greater variation is observed in velocities at a particular point for the changes of $Ra$ than $\phi$.

In general, the present analysis brings out the effects of influencing factors on the heat transfer in developing the effective choice when water-Cu nanofluid may be used.

REFERENCES
Hussain, S. H. 2010. Combined convection flow through inclined rectangular enclosure with a sliding wavy