A VIBRATION BASED INVERSE HYBRID INTELLIGENT METHOD FOR STRUCTURAL HEALTH MONITORING

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ABSTRACT
In this paper, a novel identification algorithm (hybrid intelligent system) using inverse analysis of the vibration response of a cracked cantilever beam has been proposed. The crack identification algorithm utilizes the vibration signatures of the cracked beam derived from finite element and theoretical analysis. The hybrid controller is designed to predict the crack locations and their severities by integrating the capabilities of fuzzy logic and neural network technique. The measured modal parameters along with the outputs from the fuzzy controller are the inputs to the neural segment of the hybrid system, where as final relative crack depths and final relative crack locations are the output parameters. The derived vibration parameters are used to establish series of fuzzy rules and training patterns for the fuzzy and neural controller. Finally, the reliability of the proposed crack identification algorithm is established by comparing the results obtained from the experimental analysis.

Key words: Vibration; Multiple cracks; Natural frequency; Mode shape; Fuzzy neural controller

1. INTRODUCTION
The presence of a crack in a structural member introduces a local flexibility that affects its dynamic response. Moreover, the crack will open and close in time depending on the loading conditions and vibration amplitude. The changes in dynamic characteristics can be measured and lead to an identification of the structural changes, which eventually might lead to the detection of a structural flaw. So engineers and researchers are working towards development of methodology for detection of fault in damaged structures using the changes in vibration response. An energy based method for damage identification (Mazanoglu et al., 2009) in non-uniform Euler – Bernoulli beams having open cracks using Rayleigh – Ritz approximation method has been proposed. Method has been presented to identify crack in a beam (Lee, 2009) by modeling the cracks as rotational springs. Newton-Rapson method has been adapted by him to identify the locations and sizes of the crack in a beam.

A damage assessment technique has been presented (Faverjon & Sinou 2009) for detection of size of the open crack in beams. They have used constitutive relation error updating method for identification of crack location and size of the beam. The stress intensity factor and local flexibility matrix for cracked pipes have been calculated (He et al., 2009) by dividing the cracked pipe into series of these annuli. They have described that the local flexibility matrix for cracked pipes have been calculated experimentally without calculating the Stress intensity factor. The influence of two transverse open cracks on the antiresonances of a double cracked cantilever beam (Douka et al., 2009) both analytically and experimentally have been presented. The results of experiments performed by them on Plexiglas beams for crack location and severity are in good agreement with theoretical predictions. Three different linear theories: Euler – Bernoulli, Timo Shenko and Two dimensional elasticity (Labuschagne et al., 2009) for crack detection of cantilever beams have been presented. A clonal selection programming (CSP)-based fault detection system has been developed (Gan et al., 2009) to perform induction machine fault detection and analysis. The proposed CSP-based machine fault diagnostic system has been intensively tested with unbalanced electrical faults and mechanical faults operating at different rotating speeds. A fuzzy finite element approach has been proposed (Akpan et al., 2001) for modeling smart structures with imprecise parameters.

Theories for strain energy density function with the help of stress intensity factor (Tada et al., 1973) at the crack section have been proposed to calculate the local flexibility matrix. A mobile robot navigation control system has been designed (Parhi, 2005) using fuzzy logic. Fuzzy rules embedded in the controller of a mobile robot enable it to avoid obstacles in a cluttered environment that includes other mobile robots. The use of neural network has been presented (Haykin, 1999) as a data processing tool for various applications. A neural network technique has been developed (Zubaydi et al., 2002) for identifying the damage occurrence in the side shell of a ship’s structure. The input to the network is the autocorrelation function of the vibration response of the structure. The response is obtained using a finite element model of the structure. An optimized gear fault identification system (Rafiee et al., 2009) has been developed using genetic algorithm.
(GA) to investigate the type of gear failures of a complex gearbox system using artificial neural networks (ANNs) with a well-designed structure suited for practical implementations. A crack detection method has been proposed (Zacharias et al., 2004) using an artificial neural network (ANN) trained exclusively with frequency response spectra from finite-element simulations. A method has been designed (Suresh et al., 2004) for crack detection considering the flexural vibration in a cantilever beam having transverse crack. They have computed modal frequency parameters analytically for various crack locations and depths and these parameters are used to train the neural network to identify the damage location and size. The performance of recurrent neural networks (RNNs) and neuro-fuzzy (NF) systems has been evaluated (Wang & Kanneg, 2009) using two benchmark data sets. Through comparison it is found that if an NF system is properly trained, it performs better than RNNs in both forecasting accuracy and training efficiency. The performance of the developed prognostic system is evaluated by using three test cases. Artificial Neural Network has been used (Bakhary et al., 2007) for damage detection. The developed approach is applied to detect simulated damage in a numerical steel portal frame model and also in a laboratory tested concrete slab. A neuro-fuzzy control strategy has been proposed, (Xu & Guo, 2008) in which the neural-network technique is adopted to solve time-delay problem and the fuzzy controller is used to determine the control current of MR dampers quickly and accurately. Semi-active suspension control system (Rashid et al., 2008) has been proposed using fuzzy logic and fuzzy hybrid controller for a car model. The developed car model has been described as a system of two degree of freedom. They have used MR damper for implementation of the designed control system. System based on artificial neural networks (ANN) have been proposed (Jahirul et al., 2009) to predict the brake specific fuel consumption for a four cylinder compressed natural gas fueled spark ignition internal combustion engine. They have tested the developed ANN with six different algorithms. They have used Engine speed, throttle position, fuel-air equivalence ratio and torque as input parameters while break specific fuel consumption has been used as output parameter of the developed neural system. Neuro-fuzzy analysis has been carried out (Seng et al., 2010) for hand grip assessment for patient rehabilitation. The purpose of their study is to collect the hand grip movements of the patient and distinguish them from normal persons. Neural network with back propagation algorithm has been used to design the neuro-fuzzy system.

In the current investigation, a novel approach has been developed and presented using fuzzy logic and neural network for crack prediction in a cantilever beam containing multiple transverse cracks. The vibration characteristics of the cracked beam structure have been calculated using finite element and numerical analysis. A comparison of results obtained from both the methods has been presented. An inverse analysis has been carried out using fuzzy neural controller (hybrid system) for multiple cracks detection. The results from experimental analysis have been compared with the results from theoretical, hybrid controller and FEM and a close agreement has been observed between the results.

Figure 1 Geometry of beam, (a) Cantilever beam, (b) Cross-sectional view of the beam.
2. THEORETICAL ANALYSIS

2.1 Local flexibility of a cracked beam under bending and axial loading

The presence of a transverse surface crack of depth ‘a₁’ and ‘a₂’ on beam of width ‘B’ and height ‘W’ introduces a local flexibility, which can be defined in matrix form, the dimension of which depends on the degrees of freedom. Here a 2x2 matrix is considered. A cantilever beam is subjected to axial force (P₁) and bending moment (P₂), shown in Figure 1(a), which gives coupling with the longitudinal and transverse motion. The cross sectional view of the beam is shown in Figure 1(b). The strain energy release rate at the fractured section can be written as (Tada et al., 1973);

\[
J = \frac{1}{E'}(K_{11} + K_{12})^2, \quad (1)
\]

Where \( \frac{1}{E'} = \frac{1 - \nu^2}{E'} \) (for plane strain condition); \( \frac{1}{E'} = \frac{1}{E} \) (for plane stress condition)

\( K_{11} \) and \( K_{12} \) are the stress intensity factors of mode I (opening of the crack) for load \( P_1 \) and \( P_2 \) respectively. The values of stress intensity factors from earlier studies (Tada et al., 1973) are;

\[
K_{11} = \frac{P_1}{BW} \sqrt{\pi a} \left( F_1 \left( \frac{a}{W} \right) \right), \quad K_{12} = \frac{6P_2}{BW^2} \sqrt{\pi a} \left( F_2 \left( \frac{a}{W} \right) \right)
\]

\[
\bar{C}_{11} = C_{11} \frac{BE'}{2\pi}; \quad \bar{C}_{12} = C_{12} \frac{E' BW}{12\pi} = C_{21};
\]

\[
\bar{C}_{22} = C_{22} \frac{E' BW^2}{72\pi}
\]

The local stiffness matrix can be obtained by taking the inversion of compliance matrix, i.e.

\[
K = \begin{bmatrix}
K_{11} & K_{12} \\
K_{21} & K_{22}
\end{bmatrix} = \begin{bmatrix}
C_{11} & C_{12} \\
C_{21} & C_{22}
\end{bmatrix}^{-1}
\]

Figure 2 shows the variation of dimensionless compliances to that of relative crack depth.

2.2 Analysis of vibration characteristics of the cracked beam

A cantilever beam of length ‘L’ width ‘B’ and depth ‘W’, with a crack of depth ‘a₁’ and ‘a₂’ at a distance ‘L₁’ and ‘L₂’ respectively from the fixed end is considered (shown in Figure 3).

![Figure 2](image1.png)

**Figure 2** Relative crack depth (a₁/W) vs. dimensionless compliance (ln(\( C_{sy} \))).

![Figure 3](image2.png)

**Figure 3** Front view of the cracked cantilever beam.
The displacement vector in (16) is due to the crack. Where overall flexibility matrix $C$ can be expressed as

$$C = \left( \frac{EI}{\mu} \right)^{1/2}, \quad \mu = A\rho$$

$A_i$ (i=1, 18) Constants are to be determined, from boundary conditions. The boundary conditions of the cantilever beam in consideration are:

$$\begin{align*}
\bar{u}_1(0) = 0; & \quad \bar{y}_1(0) = 0; \\
\bar{u}'_1(0) = 0; & \quad \bar{y}'_1(0) = 0;
\end{align*}$$

At the cracked section:

$$\bar{u}'_1(\beta_1) = \bar{u}'_1(\beta_2); \quad \bar{y}'_1(\beta_1) = \bar{y}'_1(\beta_2);$$

Also at the cracked section $L_1$, we have;

$$\frac{AE}{L} \frac{d}{dx} = K_{11}(u_2(L_1) - u_1(L_1))$$

Similarly, multiplying both sides of (10) by $\frac{EI}{L^2K_{22}K_{21}}$ we get,

$$M_4 \bar{y}'_1(\beta_1) + M_3 \bar{y}'_2(\beta_1) = M_3(\bar{u}_2(\beta_1) - \bar{u}_1(\beta_1))$$

$$+ M_4(\bar{y}'_1(\beta_1) - \bar{y}'_2(\beta_1))$$

(11)

Where, $M_4 = \frac{AE}{LK_{11}}, M_2 = \frac{AE}{K_{12}}, M_3 = \frac{EI}{LK_{22}},$.

Similarly at the crack section $L_2$ we can have the expression;

$$M_4 \bar{y}'_2(\beta_2) = M_3(\bar{u}_2(\beta_2) - \bar{u}_1(\beta_2))$$

$$+ M_4(\bar{y}'_1(\beta_2) - \bar{y}'_2(\beta_2))$$

(13)

The normal functions, (2) to (7) along with the boundary conditions as mentioned above, yield the characteristic equation of the system as;

$$\left| Q \right| = 0$$

(15)

This determinant is a function of natural circular frequency ($\omega$), the relative locations of the crack ($\beta_1, \beta_2$) and the local stiffness matrix ($K$) which in turn is a function of the relative crack depth ($a/W, a_1/W$). The results of the theoretical analysis for the first three mode shapes for un-cracked and cracked beam are shown in the Figure 4.

3 ANALYSIS OF CRACKED BEAM USING FINITE ELEMENT METHOD (FEM)

In the following section FEM is analyzed for vibration analysis of a cantilever cracked beam (see Figure 5). The relationship between the displacement and the forces can be expressed as;

$$\begin{align*}
\begin{bmatrix}
\bar{u}_i - u_i \\
\bar{y}_i - y_i
\end{bmatrix} &= C_{ovl} \begin{bmatrix}
u_i \\
\theta_i
\end{bmatrix}
\end{align*}$$

(16)

Where overall flexibility matrix $C_{ovl}$ can be expressed as;

$$C_{ovl} = \begin{bmatrix}
C_{11} & -C_{12} \\
-C_{21} & C_{22}
\end{bmatrix}$$

The displacement vector in (16) is due to the crack.
Figure 4a Relative amplitude vs. relative distance From the fixed end (1\textsuperscript{st} mode of vibration), $a_1/W=0.166$, $a_2/W=0.25, L_1/L=0.0625$, $L_2/L=0.3125$.

Figure 4a1 Magnified view of Figure 4a at the vicinity of the crack location $L_1/L=0.0625$.

Figure 4a2 Magnified view of Figure 4a at the vicinity of the crack location $L_2/L=0.3125$.

Figure 4b Relative amplitude vs. relative distance From the fixed end (2\textsuperscript{nd} mode of vibration), $a_1/W=0.166$, $a_2/W=0.25, L_1/L=0.0625$, $L_2/L=0.3125$.

Figure 4b1 Magnified view of Figure 4b at the vicinity of the crack location $L_1/L=0.0625$.

Figure 4b2 Magnified view of Figure 4b at the vicinity of the crack location $L_2/L=0.3125$. 
Figure 5 View of a crack beam element subjected to axial and bending forces.

Figure 4c1 Magnified view of Figure 4c at the vicinity of the crack location $L_1/L=0.0625$.

Figure 4c Magnified view of Figure 4c at the vicinity of the crack location $L_1/L=0.0625$. Relative amplitude vs. relative distance from the fixed end ($3^{rd}$ mode of vibration), $a_1/W=0.166$, $a_2/W=0.25$, $L_1/L=0.0625$, $L_2/L=0.3125$.

Figure 4c2 Magnified view of Figure 4c at the vicinity of the crack location $L_2/L=0.3125$. Relative amplitude vs. relative distance from the fixed end ($3^{rd}$ mode of vibration), $a_1/W=0.166$, $a_2/W=0.25$, $L_1/L=0.0625$, $L_2/L=0.3125$. Relative distance from fixed end.
The forces acting on the beam element for FEM analysis are shown in Figure 5.

Under this system, the flexibility matrix $C_{\text{intact}}$ of the intact beam element can be expressed as:

$$
\begin{bmatrix}
\delta_j - \delta_i \\
\theta_j - \theta_i
\end{bmatrix} = C_{\text{intact}} \begin{bmatrix}
\delta_j \\
\theta_j
\end{bmatrix}
$$

(17)

Where,

$$C_{\text{intact}} = \begin{bmatrix}
Le/EA & 0 \\
0 & Le/EI
\end{bmatrix}
$$

The displacement vector in (17) is for the intact beam.

The total flexibility matrix $C_{\text{tot}}$ of the cracked beam element can now be obtained by

$$C_{\text{tot}} = C_{\text{intact}} + C_{\text{ovl}} = \begin{bmatrix}
Le/EA + C_{11} & -C_{12} \\
-C_{21} & Le/EI + C_{22}
\end{bmatrix}
$$

(18)

Through the equilibrium conditions, the stiffness matrix $K_c$ of a cracked beam element can be obtained as (Suresh et al., 2004):

$$K_c = DC_{\text{tot}}D^T
$$

(19)

Where $D$ is the transformation matrix and expressed as:

$$D = \begin{bmatrix}
-1 & 0 \\
0 & -1 \\
1 & 0 \\
0 & 1
\end{bmatrix}
$$

The results of the finite element analysis for the first three mode shapes of the cracked beam are compared with that of the numerical analysis of the cracked beam and are presented in Figure 6.
4 ANALYSIS OF THE FUZZY PART OF FUZZY-NEURAL CONTROLLER

The fuzzy part of the fuzzy-neural controller developed has got six input parameters and four output parameters. The linguistic terms used for the inputs in the fuzzy system of the fuzzy-neural controller are as follows:

Relative first natural frequency = “fnf”; Relative second natural frequency = “snf”; Relative third natural frequency = “tnf”; Average relative first mode shape difference = “fmd”; Average relative second mode shape difference = “smd”; Average relative third mode shape difference = “tmd”.

The linguistic term used for the outputs are as follows:

Initial first relative crack location = “rcd1_init”, Initial second relative crack location = “rcd2_init”, Initial first relative crack depth = “rcd1_initial”, Initial second relative crack location = “rcl2_initial”, Initial first relative crack depth = “rcl1_initial”, Initial second relative crack depth = “rcl2_initial”.

The Membership functions names for the linguistic terms, shown in Figure 7b, used in the fuzzy inference system are described in Table 1. The membership functions are Gaussian as shown in Figure 7a.

4.1 Analysis of fuzzy mechanism of the fuzzy-neural controller

4.1.1 Fuzzy mechanism for crack detection

Based on the above fuzzy subsets, the fuzzy control rules are defined in a general form as follows:

If (fnf is fnf_i and snf is snf_j and tnf is tnf_k and fmd is fmd_l and smd is smd_m and tmd is tmd_n) then rcl1 is rcl1ijklmn and rcd1 is rcd1ijklmn (20)

and rcl2 is rcl2ijklmn and rcd2 is rcd2ijklmn

Where i=1 to 10, j=1 to 10, k=1 to 10, l=1 to 10, m=1 to 10, n=1 to 10

Because “fnf”, “snf”, “tnf”, “fmd”, “smd”, “tmd” have ten membership functions each.

From expression (20), two set of rules can be written

If (fnf is fnf_i and snf is snf_j and tnf is tnf_k and fmd is fmd_l and smd is smd_m and tmd is tmd_n) then rcl1 is rcl1ijklmn and rcd1 is rcd1ijklmn

If (fnf is fnf_i and snf is snf_j and tnf is tnf_k and fmd is fmd_l and smd is smd_m and tmd is tmd_n) then rcl1 is rcl1ijklmn and rcd1 is rcd1ijklmn

According to the usual fuzzy logic control method (Parhi, 2005), a factor \( W_{ijklmn} \) is defined for the rules as follows:

\[
W_{ijklmn} = \mu_{fnf_i} \Lambda \mu_{snf_j} \Lambda \mu_{tnf_k} \Lambda \mu_{fmd_l} \Lambda \mu_{smd_m} \Lambda \mu_{tmd_n}
\]

Where freq_i, freq_j and freq_k are the first, second and third relative natural frequencies of the cantilever beam with crack respectively; moddif_i, moddif_m and moddif_n are the average first, second and third relative mode shape differences of the cantilever beam with crack respectively. By applying the composition rule of inference (Parhi, 2005), the membership values of the relative crack location and relative crack depth, (location)_{rcd1} and (depth)_{rcd1} (v=1,2) can be computed as:

\[
\mu_{rcd1} \text{(location)} = \bigvee_{\text{depth} \in rcd1} W_{ijklmn} \mu_{rcd1} \text{(depth)}
\]

The overall conclusion by combining the outputs of all the fuzzy rules can be written as follows:

\[
\mu_{rcd1} \text{(location)} = \bigvee_{\text{depth} \in rcd1} W_{ijklmn} \mu_{rcd1} \text{(location)} \vee \ldots \vee \mu_{rcd1} \text{(location)}
\]

The crisp values of relative crack location and relative crack depth are computed using the centre of gravity method (Parhi, 2005) as:

Initial relative crack location = \[
\int \mu_{rcd1} \text{(location)} \cdot d(\text{location}) \cdot d(\text{depth})
\]

Initial relative crack depth = \[
\int \mu_{rcd1} \text{(depth)} \cdot d(\text{location}) \cdot d(\text{depth})
\]

Figure 7a Fuzzy controller

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Figure 7b1 Membership functions for relative natural frequency for first mode of vibration.

Figure 7b2 Membership functions for relative natural frequency for second mode of vibration.

Figure 7b3 Membership functions for relative natural frequency for third mode of vibration.

Figure 7b4 Membership functions for relative mode shape difference for first mode of vibration.

Figure 7b5 Membership functions for relative mode shape difference for second mode of vibration.

Figure 7b6 Membership functions for relative mode shape difference for third mode of vibration.

Figure 7b7 (a) Membership functions for relative crack depth1.

Figure 7b7 (b) Membership functions for relative crack depth2.

Figure 7b8 (a) Membership functions for relative crack location1.

Figure 7b8 (b) Membership functions for relative crack location2.
Table 1 Description of fuzzy Linguistic terms

<table>
<thead>
<tr>
<th>Membership Functions Name</th>
<th>Linguistic Terms</th>
<th>Description and range of the Linguistic terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>L1F1,L1F2,L1F3,L1F4</td>
<td>fnf 1 to 4</td>
<td>Low ranges of relative natural frequency for first mode of vibration in descending order respectively</td>
</tr>
<tr>
<td>M1F1,M1F2</td>
<td>fnf 5,6</td>
<td>Medium ranges of relative natural frequency for first mode of vibration in ascending order respectively</td>
</tr>
<tr>
<td>H1F1,H1F2,H1F3,H1F4</td>
<td>fnf 7 to 10</td>
<td>Higher ranges of relative natural frequency for first mode of vibration in ascending order respectively</td>
</tr>
<tr>
<td>L2F1,L2F2,L2F3,L2F4</td>
<td>snf 1 to 4</td>
<td>Low ranges of relative natural frequency for second mode of vibration in descending order respectively</td>
</tr>
<tr>
<td>M2F1,M2F2</td>
<td>snf 5,6</td>
<td>Medium ranges of relative natural frequency for second mode of vibration in ascending order respectively</td>
</tr>
<tr>
<td>H2F1,H2F2,H2F3,H2F4</td>
<td>snf 7 to 10</td>
<td>Higher ranges of relative natural frequencies for second mode of vibration in ascending order respectively</td>
</tr>
<tr>
<td>L3F1,L3F2,L3F3,L3F4</td>
<td>tnf 1 to 4</td>
<td>Low ranges of relative natural frequencies for third mode of vibration in descending order respectively</td>
</tr>
<tr>
<td>M3F1,M3F2</td>
<td>tnf 5,6</td>
<td>Medium ranges of relative natural frequencies for third mode of vibration in ascending order respectively</td>
</tr>
<tr>
<td>H3F1,H3F2,H3F3,H3F4</td>
<td>tnf 7 to 10</td>
<td>Higher ranges of relative natural frequencies for third mode of vibration in ascending order respectively</td>
</tr>
<tr>
<td>S1M1,S1M2,S1M3,S1M4</td>
<td>fmd 1 to 4</td>
<td>Small ranges of first relative mode shape difference in descending order respectively</td>
</tr>
<tr>
<td>M1M1,M1M2</td>
<td>fmd 5,6</td>
<td>Medium ranges of first relative mode shape difference in ascending order respectively</td>
</tr>
<tr>
<td>H1M1,H1M2,H1M3,H1M4</td>
<td>fmd 7 to 10</td>
<td>Higher ranges of first relative mode shape difference in ascending order respectively</td>
</tr>
<tr>
<td>S2M1,S2M2,S2M3,S2M4</td>
<td>smd 1 to 4</td>
<td>Small ranges of second relative mode shape difference in descending order respectively</td>
</tr>
<tr>
<td>M2M1,M2M2</td>
<td>smd 5,6</td>
<td>Medium ranges of second relative mode shape difference in ascending order respectively</td>
</tr>
<tr>
<td>H2M1,H2M2,H2M3,H2M4</td>
<td>smd 7 to 10</td>
<td>Higher ranges of second relative mode shape difference in ascending order respectively</td>
</tr>
<tr>
<td>S3M1,S3M2,S3M3,S3M4</td>
<td>tmd 1 to 4</td>
<td>Small ranges of third relative mode shape difference in descending order respectively</td>
</tr>
<tr>
<td>M3M1,M3M2</td>
<td>tmd 5,6</td>
<td>Medium ranges of third relative mode shape difference in ascending order respectively</td>
</tr>
<tr>
<td>H3M1,H3M2,H3M3,H3M4</td>
<td>tmd 7 to 10</td>
<td>Higher ranges of third relative mode shape difference in ascending order respectively</td>
</tr>
<tr>
<td>S1L1,S1L2……S1L22</td>
<td>rcl1 1 to 22</td>
<td>Small ranges of relative crack location in descending order respectively</td>
</tr>
<tr>
<td>M1L1,M1L2</td>
<td>rcl1 23,24</td>
<td>Medium ranges of relative crack location in ascending order respectively</td>
</tr>
<tr>
<td>B1L1,B1L2……B1L22</td>
<td>rcl1 25 to 46</td>
<td>Bigger ranges of relative crack location in ascending order respectively</td>
</tr>
<tr>
<td>S1D1,S1D2……S1D9</td>
<td>rcd1 1 to 9</td>
<td>Small ranges of relative crack depth in descending order respectively</td>
</tr>
<tr>
<td>M1D</td>
<td>rcd1 10</td>
<td>Medium relative crack depth</td>
</tr>
<tr>
<td>L1D1,L1D2……L1D9</td>
<td>rcd2 1 to 9</td>
<td>Small ranges of relative crack depth in descending order respectively</td>
</tr>
<tr>
<td></td>
<td>rcd2 10</td>
<td>Medium relative crack depth</td>
</tr>
<tr>
<td></td>
<td>rcd2 11 to 19</td>
<td>Larger ranges of relative crack depth in ascending order respectively</td>
</tr>
<tr>
<td>Sl.No.</td>
<td>Examples of some rules used in the fuzzy controller</td>
<td></td>
</tr>
<tr>
<td>-------</td>
<td>---------------------------------------------------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>If fnf is H1F1, snf is M2F2, tfm is M3F1, smd is H1M2, tmd is H2M4, tmd is H3M3, then rcd1 is S1D6, and rcl1 is S1L17 and rcd2 is S2D4, and rcl2 is S2L6.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>If fnf is L1F4, snf is L2F4, tfm is L3F4, smd is H1M1, tmd is H2M1, tmd is H3M2, then rcd1 is S1D2, and rcl1 is S1L17 and rcd2 is S2D1, and rcl2 is M2L2.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>If fnf is L1F3, snf is L2F4, tfm is M1M2, smd is H2M2, tmd is H3M3, then rcd1 is M1D, and rcl1 is S1L17 and rcd2 is S2D2, and rcl2 is B2L19.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>If fnf is H1F2, snf is H2F1, tfm is H3M1, smd is H2M4, tmd is H3M4, then rcd1 is S1D6, and rcl1 is S1L11 and rcd2 is S2D4, and rcl2 is M2L2.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>If fnf is M1F1, snf is L2F2, tfm is L3F3, smd is H1M1, tmd is H2M1, tmd is H3M2, then rcd1 is S1D2, and rcl1 is S1L11 and rcd2 is S2D1, and rcl2 is B2L13.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>If fnf is L1F1, snf is L2F2, tfm is M1M2, smd is M2M1, tmd is H3M4, then rcd1 is M1D, and rcl1 is S1L11 and rcd2 is S2D7, and rcl2 is M2L2.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>If fnf is L1F4, snf is L2F4, tfm is L3F4, smd is H2M1, tmd is H3M1, then rcd1 is L1D1, and rcl1 is S1L11 and rcd2 is S2D4, and rcl2 is B2L10.</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>If fnf is H1F1, snf is M2F2, tfm is M3F1, smd is H1M2, tmd is H2M2, tmd is H3M2, then rcd1 is S1D6, and rcl1 is S1L6 and rcd2 is S2D4, and rcl2 is B2L5.</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>If fnf is L1F1, snf is L2F4, tfm is L3F4, smd is M2M1, tmd is M3M2, then rcd1 is S1D2, and rcl1 is S1L6 and rcd2 is L2D1, and rcl2 is B2L5.</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>If fnf is M1F1, snf is L2F2, tfm is L3F1, smd is M2M2, tmd is H3M1, then rcd1 is S1D1, and rcl1 is S1L16 and rcd2 is S2D4, and rcl2 is B2L5.</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>If fnf is M1F1, snf is M2F1, tfm is M3F1, smd is H1M3, tmd is H2M3, tmd is H3M4, then rcd1 is S1D6, and rcl1 is S1L18 and rcd2 is S2D5, and rcl2 is M2L2.</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>If fnf is M1F1, snf is L2F1, tfm is L3F1, smd is H1M3, tmd is H2M3, then rcd1 is S1D4, and rcl1 is S1L7 and rcd2 is S2D6, and rcl2 is S2L6.</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>If fnf is M1F2, snf is M2F1, tfm is M3F1, smd is H2M1, tmd is H3M2, then rcd1 is S1D4, and rcl1 is S1L11 and rcd2 is S2D4, and rcl2 is M2L2.</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>If fnf is H1F2, snf is H2F1, tfm is H1M4, smd is H2M2, tmd is H3M1, then rcd1 is S1D7, and rcl1 is S1L17 and rcd2 is S2D6, and rcl2 is B2L16.</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>If fnf is M1F1, snf is L2F1, tfm is L3F2, smd is S1M1, tmd is S2M2, tmd is H3M1, then rcd1 is S1D2, and rcl1 is S1L11 and rcd2 is S2D6, and rcl2 is B2L10.</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>If fnf is L1F4, snf is L2F4, tfm is H1M2, smd is S2M1, tmd is H3M2, then rcd1 is L1D1, and rcl1 is S1L17 and rcd2 is S2D5, and rcl2 is M2L2.</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>If fnf is M1F1, snf is L2F3, tfm is L3F1, smd is S1M2, tmd is M2M1, tmd is S3M1, then rcd1 is S1D6, and rcl1 is S1L12 and rcd2 is M2D, and rcl2 is M2L1.</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>If fnf is L1F1, snf is L2F1, tfm is L3F1, smd is H1M2, tmd is H3M2, then rcd1 is S1D2, and rcl1 is S1L12 and rcd2 is S2D4, and rcl2 is B2L13.</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>If fnf is H1F2, snf is H2F1, tfm is H1M2, smd is S2M2, tmd is H3M1, then rcd1 is S1D4, and rcl1 is S1L5 and rcd2 is S2D6, and rcl2 is B2L6.</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>If fnf is L1F3, snf is L2F4, tfm is S3M1, smd is S2M2, tmd is S3M3, then rcd1 is L1D1, and rcl1 is S1L5 and rcd2 is S2D2, and rcl2 is B2L5.</td>
<td></td>
</tr>
</tbody>
</table>
4.2 Analysis of the neural part of fuzzy-neural controller

In the fuzzy-neural controller, the fuzzy segment will be inherited from the section-4. The fuzzy segment in the fuzzy-neural controller will give the intermittent result for initial relative crack depths and initial relative crack locations. The neural segment of the fuzzy-neural controller has ten inputs such as intermittent relative crack depths and relative crack locations obtained from the fuzzy segment along with relative deviation of first three natural frequencies and first three average relative mode shape difference. The output from the fuzzy-neural controller is the final result for relative crack depths and relative crack locations. The analysis of the neural network used in the fuzzy-neural controller is given below. The neural network used in the fuzzy-neural controller is a back propagation neural controller has been developed for detection of the relative crack locations and relative crack depths (Figure 8). The neural network has got ten input parameters and four output parameters. The inputs to the neural segment of the fuzzy-neural controller are as follows;

Relative first natural frequency = “fnf”;
Relative second natural frequency = “snf”;
Relative third natural frequency = “tnf”;
Average relative first mode shape difference = “fmd”;
Average relative second mode shape difference = “smd”;
Average relative third mode shape difference = “tmd”.
Initial first relative crack location = “rcl1_initial”;
Initial first relative crack depth = “rcd1_initial”;
Initial second relative crack location = “rcl2_initial”;
Initial first relative crack depth = “rcd2_initial”.

The final outputs from the fuzzy-neural controller are;
Initial first relative crack location = “rcl1_final”, final first relative crack depth = “rcd1_final”,
Initial second relative crack location = “rcl2_final”, final first relative crack depth = “rcd2_final”.

The back propagation neural network used in fuzzy-neural controller has got ten layers (i.e. input layer, output layer and eight hidden layers). The neurons associated with the input and output layers are ten and four respectively. The neurons associated in the eight hidden layers are twelve, thirty-six, fifty, one hundred fifty, three hundred, one hundred fifty, fifty and eight respectively. The input layer neurons represent relative deviation of first three natural frequencies and first three average relative mode shape difference along with the four outputs from the fuzzy segment. The output layer neurons represent final relative crack locations and final relative crack depths. The neurons are taken in order to give the neural network a diamond shape (Figure 8).

4.2.1 Analysis of neural controller mechanism of the fuzzy-neuro controller

The neural network of the fuzzy-neural controller used in the ten-layer perceptron (Haykin, 1999). The chosen number of layers was found empirically to facilitate training. The input layer has ten neurons, three for first three relative natural frequencies and three for first three average relative mode shape difference and the other four are the outputs from the fuzzy segment. The output layer has four neurons, which represent final relative crack locations and final relative crack depths. The first hidden layer has 12 neurons, the second hidden layer has 36 neurons, the third hidden layer has 50 neurons, the fourth hidden layer has 150 neurons, the fifth hidden layer has 300 neurons, the sixth hidden layer has 150 neurons, the seventh hidden layer has 50 neurons and the eighth hidden layer has 8 neurons. These numbers of hidden neurons are also found empirically. Figure 8 depicts the fuzzy-neuro controller with its input and output signals. The neural network is trained with 800 patterns representing typical scenarios, some of which are depicted in Table 3. For example, from Table 3 when the relative deviation of first three natural frequencies, first three average relative mode shapes and the initial relative crack depths, initial relative crack locations are 0.9974, 0.9997, 0.9995, 0.0011, 0.9852, 0.2314, 0.161, 0.20, 0.413, 0.45 respectively then the final relative crack locations and final relative crack depths are 0.166, 0.25 and 0.418, 0.50 respectively. The neural network is trained to give outputs such as relative crack depths and relative crack locations. During training and during normal operation, the input patterns fed to the neural segment of the fuzzy-neural controller comprise the following components:

\[
y^{(1)}_1 = \text{relative deviation of first natural frequency} (25a)
\]

\[
y^{(2)}_1 = \text{relative deviation of second natural frequency} (25b)
\]

\[
y^{(3)}_1 = \text{relative deviation of third natural frequency} (25c)
\]

\[
y^{(4)}_1 = \text{average relative deviation of first mode shape} (25d)
\]

\[
y^{(5)}_1 = \text{average relative deviation of second mode shape} (25e)
\]

\[
y^{(6)}_1 = \text{average relative deviation of third mode shape} (25f)
\]

\[
y^{(7)}_1 = \text{initial first relative crack location} (25g)
\]

\[
y^{(8)}_1 = \text{initial first relative crack depth} (25h)
\]

\[
y^{(9)}_1 = \text{initial second relative crack location}(25i)
\]

\[
y^{(10)}_1 = \text{initial second relative crack depth} (25j)
\]

These input values are distributed to the hidden neurons which generate outputs given by (Haykin, 1999):

\[
y^{(lay)}_j = f(V^{(lay)}_j) (26)
\]
### Table 3  Examples of the training patterns for the hybrid intelligent Controller

<table>
<thead>
<tr>
<th>Relative first natural frequency “fnf”</th>
<th>Relative second natural frequency “snf”</th>
<th>Relative third natural frequency “tnf”</th>
<th>Average Relative first mode shape difference “fmd”</th>
<th>Average Relative second mode shape difference “smd”</th>
<th>Average Relative third mode shape difference “tmd”</th>
<th>Initial first crack depth rod1_initial</th>
<th>Initial second crack depth rod2_initial</th>
<th>Initial first crack location rcl1_initial</th>
<th>Initial second crack location rcl2_initial</th>
<th>Final relative 1&quot; crack depth “rod1”</th>
<th>1&quot; crack location “rcl1”</th>
<th>2&quot; crack depth “rod2”</th>
<th>2&quot; crack location “rcl2”</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9992</td>
<td>0.9977</td>
<td>0.9975</td>
<td>0.3721</td>
<td>0.3416</td>
<td>0.2614</td>
<td>0.413</td>
<td>0.22</td>
<td>0.161</td>
<td>0.47</td>
<td>0.417</td>
<td>0.26</td>
<td>0.166</td>
<td>0.52</td>
</tr>
<tr>
<td>0.9974</td>
<td>0.9997</td>
<td>0.9995</td>
<td>0.0011</td>
<td>0.9852</td>
<td>0.2314</td>
<td>0.161</td>
<td>0.20</td>
<td>0.413</td>
<td>0.45</td>
<td>0.166</td>
<td>0.25</td>
<td>0.418</td>
<td>0.50</td>
</tr>
<tr>
<td>0.9980</td>
<td>0.9964</td>
<td>0.9883</td>
<td>0.0023</td>
<td>0.0333</td>
<td>0.0132</td>
<td>0.22</td>
<td>0.23</td>
<td>0.411</td>
<td>0.72</td>
<td>0.27</td>
<td>0.28</td>
<td>0.416</td>
<td>0.77</td>
</tr>
<tr>
<td>0.9975</td>
<td>0.9993</td>
<td>0.9981</td>
<td>0.001</td>
<td>0.0046</td>
<td>0.0862</td>
<td>0.21</td>
<td>0.120</td>
<td>0.162</td>
<td>0.371</td>
<td>0.26</td>
<td>0.125</td>
<td>0.167</td>
<td>0.376</td>
</tr>
<tr>
<td>0.9994</td>
<td>0.9965</td>
<td>0.9968</td>
<td>0.0012</td>
<td>0.0013</td>
<td>0.0082</td>
<td>0.164</td>
<td>0.372</td>
<td>0.22</td>
<td>0.622</td>
<td>0.169</td>
<td>0.377</td>
<td>0.27</td>
<td>0.627</td>
</tr>
<tr>
<td>0.9992</td>
<td>0.9977</td>
<td>0.9975</td>
<td>0.3826</td>
<td>0.2359</td>
<td>0.2311</td>
<td>0.412</td>
<td>0.121</td>
<td>0.330</td>
<td>0.872</td>
<td>0.417</td>
<td>0.126</td>
<td>0.335</td>
<td>0.875</td>
</tr>
<tr>
<td>0.9983</td>
<td>0.9997</td>
<td>0.9986</td>
<td>0.002</td>
<td>0.0034</td>
<td>0.0809</td>
<td>0.163</td>
<td>0.22</td>
<td>0.21</td>
<td>0.46</td>
<td>0.168</td>
<td>0.27</td>
<td>0.26</td>
<td>0.51</td>
</tr>
<tr>
<td>0.9997</td>
<td>0.9959</td>
<td>0.9971</td>
<td>0.0022</td>
<td>0.0021</td>
<td>0.0072</td>
<td>0.162</td>
<td>0.120</td>
<td>0.161</td>
<td>0.870</td>
<td>0.167</td>
<td>0.125</td>
<td>0.166</td>
<td>0.875</td>
</tr>
<tr>
<td>0.9988</td>
<td>0.9858</td>
<td>0.9887</td>
<td>0.0075</td>
<td>0.0077</td>
<td>0.0292</td>
<td>0.331</td>
<td>0.372</td>
<td>0.47</td>
<td>0.621</td>
<td>0.336</td>
<td>0.377</td>
<td>0.52</td>
<td>0.626</td>
</tr>
<tr>
<td>0.9987</td>
<td>0.9993</td>
<td>0.9996</td>
<td>0.0092</td>
<td>0.0027</td>
<td>0.0036</td>
<td>0.162</td>
<td>0.121</td>
<td>0.21</td>
<td>0.370</td>
<td>0.167</td>
<td>0.126</td>
<td>0.26</td>
<td>0.375</td>
</tr>
</tbody>
</table>
Figure 8 Fuzzy-neural system for fault detection
5. EXPERIMENTAL SET-UP
Experiments are performed to determine the natural frequencies and mode shapes for different crack depths on Aluminum beam specimen (800 x 38 x 6mm). The experimental set-up is shown in Figure 9. The amplitude of transverse vibration at different locations along the length of the Aluminum beam is recorded by positioning the vibration pick-up and tuning the vibration generator at the corresponding resonant frequencies. The results for first three modes are plotted in Figure 10. Corresponding numerical and FEM results are also presented in the same graph for comparison.

6. DISCUSSION
The first three relative natural frequencies and first three average relative mode shape difference of a cracked cantilever beam has been evaluated using theoretical and finite element analyses. The beam is considered to be made of aluminum with rectangular section (38mmx6mmx800mm).

On the basis of theory used in section 2 (theoretical analysis) and section 3 (finite element analysis) the vibration signatures for first three modes of the cracked cantilever beam with several crack depths and crack locations are calculated. The relation between relative beam length and relative amplitude, for first three modes of vibration are shown graphically in Figure 4. The comparisons of modal parameters of the cracked beam from theoretical and finite element analysis are given in Figure 6 and are found to be close to each other. The schematic diagram of the fuzzy controller and the membership functions with the linguistic terms are shown in Figure 7(a) and Figure 7(b) respectively. The measured modal parameters are used to design fuzzy linguistic terms and fuzzy rules. The fuzzy linguistic terms and some of the fuzzy rules used for the designing of the fuzzy controller are presented in Table 1 and Table 2 respectively. Table 3 represents some of the training patterns for the neural segment of the hybrid system with the outputs from the hybrid controller. Figure 8 expresses the full view of the hybrid intelligent system.

Figure 9 Schematic block diagram of Experimental Set-up

1. Vibration pick-up
   (Accelerometer)
2. Vibration Analyzer
   (PULSE lite type 3560L)
3. Vibration indicator with software
   (PULSE lab shop software)
4. Distribution box
5. Power supply
6. Function Generator
7. Power amplifier
8. Vibration Exciter
9. Cantilever beam Specimen
Table 4  Comparison of results between fuzzy neural controller, numerical, FEM analysis and experimental setup.

| Relative first natural frequency “fnf” | Relative second natural frequency “snf” | Relative third natural frequency “tnf” | Average Relative first mode shape “fmd” | Average Relative second mode shape “smd” | Average Relative third mode shape “tmd” | Fuzzy Neural Controller final relative | FEM relative1 first crack depth “rcd1” | FEM relative1 crack location “rcl1” | Numerical relative1 crack depth “rcd1” | Numerical relative1 crack location “rcl1” | Experimental relative1 crack depth “rcd1” | Experimental relative1 crack location “rcl1” |
|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|----------------------------------------|
| 0.9992 0.9977 0.9975 0.3721 0.3416 | 0.2641 0.417 0.26 0.166 0.52 | 0.414 0.24 0.163 0.49 0.409 0.21 0.157 0.46 0.419 0.28 0.170 0.54 | 0.9974 0.9997 0.9995 0.0011 0.9852 | 0.2314 0.166 0.25 0.418 0.50 | 0.161 0.21 0.414 0.45 0.159 0.18 0.411 0.42 0.169 0.27 0.420 0.52 | 0.9980 0.9964 0.9883 0.0023 0.0333 | 0.0132 0.27 0.28 0.416 0.77 | 0.24 0.24 0.413 0.74 0.21 0.21 0.409 0.71 0.29 0.29 0.418 0.79 | 0.9975 0.9993 0.9981 0.001 0.0046 | 0.0862 0.26 0.125 0.167 0.376 0.21 0.121 0.163 0.373 0.19 0.118 0.160 0.370 0.28 0.127 0.169 0.378 | 0.9994 0.9965 0.9968 0.0012 0.0013 | 0.0082 0.169 0.377 0.27 0.627 0.163 0.373 0.24 0.624 0.158 0.369 0.20 0.621 0.171 0.379 0.29 0.629 | 0.9992 0.9977 0.9975 0.3826 0.2359 | 0.2311 0.417 0.126 0.335 0.875 0.412 0.124 0.331 0.871 0.409 0.121 0.328 0.868 0.419 0.128 0.337 0.877 | 0.9983 0.9997 0.9986 0.002 0.0034 | 0.0809 0.168 0.27 0.26 0.51 0.162 0.23 0.24 0.49 0.157 0.22 0.22 0.46 0.169 0.29 0.28 0.53 | 0.9997 0.9959 0.9971 0.0022 0.0021 | 0.0072 0.167 0.125 0.166 0.875 0.162 0.121 0.161 0.871 0.159 0.118 0.159 0.868 0.169 0.127 0.168 0.877 | 0.9988 0.9858 0.9887 0.0075 0.0077 | 0.0292 0.336 0.377 0.52 0.626 0.331 0.374 0.47 0.622 0.328 0.371 0.44 0.619 0.338 0.379 0.54 0.628 | 0.9987 0.9993 0.9996 0.0092 0.0027 | 0.0036 0.167 0.126 0.26 0.375 0.163 0.123 0.24 0.373 0.159 0.119 0.21 0.369 0.169 0.128 0.28 0.377 |
Table 4 presents a comparison of results for different crack depths and crack locations of the cracked beam obtained from theoretical, finite element, fuzzy neural controller and experimental analysis. The reliability of the identification algorithm is established using the data obtained from the experimental analysis. (Figure 9). A comparison of results from experimental, finite element method and theoretical analysis (Figure 10) for the cracked cantilever beam are presented. The mean relative error analysis of the data presented in Table 4 have yielded the following results: Mean relative error (fuzzy neural controller) = 4.2%, Mean relative error (Experimental) = 6.3%, Mean relative error (numerical) = 3.5%.

7. CONCLUSIONS
The following conclusions can be drawn from the different analyses carried out for cracks identification.
- From the analysis, it has been observed that both crack locations and crack depths have noticeable effects on the modal parameters of the cracked beam.
- The hybrid intelligent controller is developed with the computed values of modal parameters of the cracked beam with various crack depths and crack locations as input parameters and final relative crack depths and final relative crack locations as output parameters.
- The authenticity of the hybrid system can be verified from the predicted values of the crack locations and depths by comparing the results from experimental analysis.
• This modular fuzzy-neural architecture can be used as a non-destructive procedure for health monitoring of structures.
• Evolution algorithm can also be used in future to develop hybrid system for easy diagnosis of faults in dynamically vibrating structures.

REFERENCES

APPENDIX (A):
Where expressions for $F_1$ and $F_2$ are as follows

$$F_1 \left( \frac{a}{W} \right)_i = \left( \frac{2W}{\pi a} \tan \left( \frac{\pi a}{2W} \right) \right)^{0.5}$$

\[
\left\{ \begin{array}{l}
0.752 + 2.02 \left( \frac{a}{W} \right) + 0.37 \left( 1 - \sin \left( \frac{\pi a}{2W} \right) \right)^{3/2} \\
\cos \left( \frac{\pi a}{2W} \right)
\end{array} \right.
\]  

$$F_2 \left( \frac{a}{W} \right)_i = \left( \frac{2W}{\pi a} \tan \left( \frac{\pi a}{2W} \right) \right)^{0.5}$$

\[
\left\{ \begin{array}{l}
0.923 + 0.199 \left( 1 - \sin \left( \frac{\pi a}{2W} \right) \right)^4 \\
\cos \left( \frac{\pi a}{2W} \right)
\end{array} \right.
\]  

Let $U_i$ be the strain energy due to the crack. Then from Castigliano’s theorem, the additional displacement along the force $P_i$ is:

\[
u_i = \frac{\partial U_i}{\partial P_i}
\]  

(A3)
The strain energy will have the form,

$$U_t = \int_0^a \frac{\partial U_t}{\partial a} \, da = \int_0^a J \, da$$  \hspace{1cm} (A4)$$

Where $J = \frac{\partial U_t}{\partial a}$ is the strain energy density function.

From (1) and (2), thus we have

$$u_i = \frac{\partial}{\partial a} \left[ \int_0^a J(a) \, da \right]$$  \hspace{1cm} (A5)$$

The flexibility influence co-efficient $C_{ij}$ will be, by definition

$$C_{ij} = \frac{\partial u_i}{\partial P_j} = \frac{\partial^2}{\partial P_i \partial P_j} \left[ \int_0^a J(a) \, da \right]$$  \hspace{1cm} (A6)$$

and can be written as

$$C_{ij} = \frac{B W}{E} \frac{\partial^2}{\partial P_i \partial P_j} \left[ (K_{11} + K_{12})^2 \, d \xi \right]$$  \hspace{1cm} (A7)$$

From (7), calculating $C_{11}, C_{12} (=C_{21})$ and $C_{22}$ we get

APPENDIX (B):

Where

$$V^{[lay]}_j = \sum_i W^{[lay]}_{ji} y^{[lay-1]}$$  \hspace{1cm} (B1)$$

lay = layer number (2 or 9)

j = label for jth neuron in hidden layer ‘lay’

i = label for ith neuron in hidden layer ‘lay-1’

$W^{[lay]}_{ji}$ = weight of the connection from neuron i in layer ‘lay-1’ to neuron j in layer ‘lay’

f (.) = activation function, chosen in this work as the hyperbolic tangent function:

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$  \hspace{1cm} (B2)$$

During training, the network output $\theta_{actual,n}$ for $i=1$ to 4j may differ from the desired output $\theta_{desired,n}$, $i=1$ to 4j, as specified in the training pattern presented to the network. A measure of the performance of the network is the instantaneous sum-squared difference between $\theta_{desired,n}$ and $\theta_{actual,n}$ for the set of presented training patterns:

$$Err = \frac{1}{2} \sum_{all \ training \ patterns} \left( \theta_{desired,n} - \theta_{actual,n} \right)^2$$  \hspace{1cm} (B3)$$

Where $\theta_{actual,n}$ (n=1) represents relative crack location (“rc11”)

$\theta_{actual,n}$ (n=2) represents relative crack depth (“rcd1”)

$\theta_{actual,n}$ (n=3) represents relative crack location (“rc12”)

$\theta_{actual,n}$ (n=4) represents relative crack depth (“rcd2”)

The error back propagation method is employed to train the network (Haykin, 1999). This method requires the computation of local error gradients in order to determine appropriate weight corrections to reduce error. For the output layer, the error gradient $\delta^{[10]}$ is:

$$\delta^{[10]} = f'(V^{[lay]}_j) \left( \theta_{desired,n} - \theta_{actual,n} \right)$$  \hspace{1cm} (B4)$$

The local gradient for neurons in hidden layer [lay] is given by:

$$\delta^{[lay]}_j = f'(V^{[lay]}_j) \left( \sum_k \delta^{[lay+1]}_k W^{[lay+1]}_{kj} \right)$$  \hspace{1cm} (B5)$$

The synaptic weights are updated according to the following expressions:

$$W_{ji}(t+1) = W_{ji}(t) + \Delta W_{ji}(t+1)$$  \hspace{1cm} (B6)$$

and

$$\Delta W_{ji}(t+1) = \alpha \Delta W_{ji}(t) + \eta \delta^{[lay]}_j y^{[lay-1]}$$  \hspace{1cm} (B7)$$

Where

$\alpha$ = momentum coefficient (chosen empirically as 0.2 in this work)

$\eta$ = learning rate (chosen empirically as 0.35 in this work)

t = iteration number, each iteration consisting of the presentation of a training pattern and correction of the weights.

The final output from the neural network is:

$$\theta_{actual,n} = f(V^{[10]}_n)$$  \hspace{1cm} (B8)$$

$$V^{[10]}_n = \sum_i W^{[10]}_{ni} y^{[9]}_i$$  \hspace{1cm} (B9)$$

where