MIXED CONVECTION THROUGH A LID-DRIVEN AIR–FILLED SQUARE CAVITY WITH A HOT WAVY WALL

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ABSTRACT

Numerical simulation of mixed convection air flow through a lid-driven square cavity is studied. Physical problem consists of a square lid-driven cavity filled with an air. The top and bottom surfaces are thermally insulated, while the right wall is considered wavy and it is maintained at isothermal hot temperature with different undulations and the left wall is a lid–driven which is maintained at isothermal cold temperature. The problem considered deals with a two-dimensional internal, laminar, lid-driven flow over a square air-filled cavity. The two-dimensional mass, momentum and energy equations with Boussinesq approximation are discretised using the control volume based on finite-volume method with collocated variable arrangement. The flow characteristics are evaluated for Prandtl number at 0.71 and the amplitude of the wavy surface varies from 0 to 0.075. This study included computations for cavities at Grashof number, 10^4 and Richardson number ranging 0.01, 1.0 and 10 with one, two and three undulations. The results are presented in terms of isotherms and streamlines plots as well as the local heat flux is computed with different number of undulations. It is found that the two-dimensional flow is characterized by the vortices that are initiated by the existence of buoyancy effect. The results explain that the flow field is dominated by a rotating one cell filling the cavity. The results showed a good agreement with other published results.

Keywords: Mixed Convection, Square Cavity, Lid-Driven, Laminar Flow, Wavy Surface.

NOMENCLATURE

\( A \) Dimensionless amplitude of the wavy right side surface
\( g \) Gravitational acceleration, (m/s^2)
\( Gr \) Grashof number
\( H \) Side length of the cavity, (m)
\( k \) Thermal conductivity of fluid, (W / m.°C)
\( Nu_{av} \) Average Nusselt number
\( P \) Dimensionless pressure
\( p \) Pressure, (N/m^2)
\( Pr \) Prandtl number
\( Re \) Reynolds number.
\( Ri \) Richardson number.
\( S \) Total chord length of the wavy surface,(m)
\( s \) Coordinate along the wavy surface,(m)
\( T \) Temperature, (°C)
\( T_h \) Temperature of the hot wall,(°C)
\( T_c \) Temperature of the cold wall,(°C)
\( U \) Dimensionless velocity component in x-direction
\( u \) Velocity component in x-direction, (m/s)
\( V \) Dimensionless velocity component in y-direction
\( V_{lid} \) Sliding left wall velocity, (m/s)
\( v \) Velocity component in y-direction, (m/s)
\( X \) Dimensionless Coordinate in horizontal direction
\( x \) Cartesian coordinate in horizontal direction, (m)
\( Y \) Dimensionless Coordinate in vertical direction
\( y \) Cartesian coordinate in vertical direction, (m)
\( \alpha \) Thermal diffusivity, (m^2/s)
\( \beta \) Volumetric coefficient of thermal expansion, (K^-1)
\( \lambda \) Number of undulations.
\( \theta \) Dimensionless temperature
\( \nu \) Kinematic viscosity of the fluid, (m^2/s)
\( \rho \) Density of the fluid, (kg/m^3)

1. INTRODUCTION

The process of heat transfer by mixed convection in lid-driven cavities occurs in many industrial and technical applications which include cooling of electronic equipments, solar central receivers, lakes and reservoirs, nuclear power plants and heat exchangers placed in a low velocity environment (Arpaciand Larsen, 1984). The lid-driven closed cavities mechanically driven by tangentially moving walls represents a basic problem in convection heat transfer. In the classical configuration, one of the horizontal walls or both are moving either steadily or in a time-dependent manner tangentially to itself. In the present work, the classical configuration is modified to make one of the side walls are moving with a constant velocity. However, Mixed convection in cavities caused by buoyancy forces have been studied deeply in the literatures. Das and Mahmud (2003) and Hasanuzzaman et al. (2007) studied numerically free convection in a cavity which has a two insulated vertical walls and a two horizontal isothermal wavy walls. They concluded that the average Nusselt number was affected by the wavy shape of walls. Adjloot et al. (2002) studied numerically laminar natural convection flow in an inclined cavity which has a heated wavy wall. The results explained that the heated wavy wall causes a decrease of heat transfer rate as compared with the square cavity. Prasad and Koseff (1996) studied the mixed convection...
heat transfer process within a recirculating flow in an insulated lid-driven cavity of rectangular cross section. The computed mean heat flux values over the entire lower boundary were analyzed to produce Nusselt number and Stanton number correlations. Hasanuzzaman et al. (2009) investigated convection heat transfer in rectangular cavity. Oztop and Varol (2009) performed a numerical study to obtain combined convection field in an inclined porous lid-driven enclosures heated from one wall with a non-uniformly heater. It was observed that flow field, temperature distribution and heat transfer are affected by inclination angle of the enclosure. Al-Amiri et al. (2007) examined the momentum and energy transport processes in a lid-driven cavity with a wavy bottom surface. The cavity is exposed under a vertical temperature gradient by subjecting the bottom wall to a relatively higher temperature than the top lid. The results are shown in terms of streamlines and isotherms for various considered dimensionless parameters. Also, the implications of the dimensionless parameters are also investigated on the Nusselt number predictions. Sharif (2007) investigated laminar mixed convection processes in shallow two-dimensional rectangular cavities at three different Richardson numbers representing the dominating forced convection, mixed convection, and dominating natural convection using the FLUENT commercial code. The effects of inclination of the cavity on the resulting convection processes are also investigated. Mohammed (2008) made an experimental study of assisted and opposed air flows through a vertical circular tube under uniform wall heat flux boundary condition to investigate the effects of flow direction and the tube inclination angles on the heat transfer characteristics for laminar mixed convection heat transfer. He concluded that for opposed flow, the variation of the surface temperature was found to be strongly dependent on Re and Gr numbers. Aounallah et al. (2007) investigated numerically the turbulent natural convection of air flow in a confined cavity with two differentially heated side walls up to Rayleigh number of $10^{12}$. A correlation of the mean Nusselt number function of the Rayleigh number was proposed for the range of Rayleigh numbers of $10^9$–$10^{12}$. Dalal and Das (2005) numerically investigated natural convection in two-dimensional enclosure with three flat and one wavy wall. One wall was having a sinusoidal temperature profile. Other three walls including the wavy wall are maintained at constant cold temperature. The results obtained showed that the angle of inclination affects the flow and heat transfer rate in the cavity and the trend of local Nusselt number was wavy. Prasad and Koseff (2003) investigated hydrodynamic and a thermal behavior of fluid inside a wavy walled enclosure consists of two wavy and two straight walls. Calculated results for Nusselt number are compared with the available references. Basak et al. (2009) investigated numerically the influence of linearly heated side wall or cooled right wall on mixed convection lid-driven flows in a square cavity. They concluded that the average Nusselt number at the bottom and right walls are strong functions of Grashof number at larger Prandtl numbers while at the left wall it was a weaker function of Grashof number. The present work is based on the previous work by Al-Amiri et al. (2007) but in their work attention has been considered to the problem of mixed convection flow and heat transfer in a lid-driven cavity that is heated from a wavy bottom surface. The present work deals with same cavity but it is heated from a wavy right side wall rather than the bottom surface. In this work the numerical investigation of mixed convection air flow through a lid-driven square cavity is studied. Physical problem consists of a square lid-driven cavity filled with an air. The top and bottom surfaces are thermally insulated, while the right side wall is considered wavy and it is maintained at isothermal hot temperature with different undulations. The left side wall is considered lid-driven at constant speed and it is maintained at isothermal cold temperature. The two-dimensional flow is characterized using a finite volume scheme. In this work, the governing equations are solved numerically using a finite volume method. The flow characteristics are evaluated for Prandtl number at 0.71 and wavy surface amplitude varies from 0 to 0.075. This study included computations for cavities at Grashof number, $10^9$ and Richardson number ranging 0.01, 1.0 and 10 with one, two and three undulations. The results are presented in terms of isotherms and stream contours graphs. Also the local heat flux is computed with a different number of undulations and amplitudes.

2. PROBLEM DESCRIPTION AND THE MATHEMATICAL ANALYSIS

The studied case is a square air–filled cavity which is considered insulated at the top and bottom walls. The right side wall of the cavity is considered wavy and kept at a uniform hot temperature ($T_h$) while the other left side wall is a lid-driven at a constant speed ($V_{lid}$) upwards and is kept at a uniform cold temperature ($T_c$). The configuration under consideration is shown schematically in Figure (1). The square cavity has a length ($H$) and the working fluid is chosen as air with Prandtl number, $Pr = 0.71$. The flow described by continuity, momentum and energy equations. These equations are written in a dimensionless form by dividing all dependent and independent variables by suitable constant terms. The solution is obtained using a finite volume method and the following assumptions are considered:-

1. The flow is considered laminar, two-dimensional and steady state.

2. The properties of the fluid are assumed constant except for the density change which is solved according to Boussinesq approximation.

3. The fluid inside the cavity is assumed Newtonian while viscous effects are negligible.

The flow and thermal fields inside the cavity are described by the Navier-Stokes and the energy equations, respectively. The governing equations are transformed into dimensionless forms under the following non-dimensional variables [Amiri et al., 2007]:

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\[ \theta = \frac{T - T_c}{T_h - T_c}, X = \frac{x}{H}, Y = \frac{y}{H} \text{ and } (U, V) = \left( \frac{u}{V_{Lid}}, \frac{v}{V_{Lid}} \right) \]

Where X and Y are the dimensionless horizontal and vertical axes respectively, u and v being the dimensional horizontal and vertical velocity components and \( \theta \) is the dimensionless temperature. The dimensionless forms of the steady governing equations are expressed in the following forms [Amiri et al., 2007]:

\[ U = V = \frac{\partial \theta}{\partial Y} = 0 \]

\[ U = 0, V = V_{Lid} , \theta = 0 \]

\[ U = V = \frac{\partial \theta}{\partial Y} = 0 \]

Figure 1 Configuration diagram of the square cavity for the hot wavy right side wall with three undulations \((A=0.05)\).  

\[ \frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \]

\[ U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{1}{Re} \frac{\partial P}{\partial X} + \frac{1}{Re} \left( \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} \right) \]

\[ \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{1}{Re} \frac{\partial P}{\partial Y} + \frac{1}{Re} \left( \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} \right) \]

\[ U \frac{\partial \theta}{\partial X} + V \frac{\partial \theta}{\partial Y} = \frac{1}{Re Pr} \left( \frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} \right) \]

where \( Pr \) is the Prandtl number and \( Re = V_{Lid} H / \nu \) is the Reynolds number. The strength of the imposed temperature gradient is given by the presentation of the Grashof number \((Gr)\) which is defined as [Sharif, 2007]:

\[ Gr = \frac{g \beta (T_h - T_c) H^3}{\nu^2} \]

Where \( \beta \) is the volumetric coefficient of thermal expansion and \( g \) is the gravitational acceleration. The ratio \( Gr/Re^2 \) is the mixed convection parameter and is called the Richardson number \((Ri)\) and is a measure of the relative strength of the natural convection and forced convection for a particular problem. If \( Ri<<1 \) then forced convection is dominant while if \( Ri>>1 \) then natural convection is dominant. For problems with \( Ri\sim1 \) then the natural convection effects are comparable to the forced convection effects. Hence the second right hand side term in the momentum equation (4) can be expressed as the Richardson number \((Ri)\) which is defined by (Aounallah et al., 2007):

\[ Ri = \frac{Gr}{Re^2} \]

The range of Richardson number which is considered in the present work is taken as the same range which was considered by Al-Amiri et al.,(2007) as 0.01,1.0 and 10 respectively just for the purpose of comparison. The Grashof number and the Prandtl number are taken as \( 10^4 \) and 0.71 respectively. The shape of the hot wavy right side wall surface is assumed to obey the following profile [Amiri et al., 2007]:

\[ Y = A(1 - \cos(2\lambda \pi X)) \]

Figure 2 Computational representation of two-dimensional grid in a lid-driven square cavity flow with non-uniform and non-orthogonal distributions for three undulations \((A=0.05)\).  

Where, \( \lambda \) is the undulations number and \( A \) is the dimensionless wavy surface amplitude. The local Nusselt number \((Nu)\) is obtained as (Sharif, 2007):

\[ Nu = \left[ \frac{\partial \theta}{\partial N} \right]_{Y=f(x)} \]

Where the gradient term \( \frac{\partial \theta}{\partial N} \) is the temperature gradient normal to the hot wavy wall. The average Nusselt number \((Nu_{av})\) is obtained by integrating the local Nusselt number along the right wavy side wall surface and it is defined by (Sharif, 2007):

\[ Nu_{av} = \frac{1}{L} \int_0^L Nu \, ds \]
where \((S)\) is the wavy surface total length and \((s)\) is the coordinate along the wavy surface.

**Boundary Conditions**

The boundary conditions which are used in the present study can be arranged as follows:-

1. The top and bottom walls are thermally insulated so:
   \[
   Y = 0 \quad \frac{\partial u}{\partial Y} = 0 \quad \text{and} \quad U = V = 0
   \]
   \[
   Y = 1 \quad \frac{\partial u}{\partial Y} = 0 \quad \text{and} \quad U = V = 0
   \]

2. The left side wall is sliding at a constant velocity and it is kept at a uniform cold temperature, so:
   \[
   \text{at} X = 0 \quad \theta = 0, \quad U = 0 \quad \text{and} \quad V = V_{Lid}
   \]

3. The right side wall is considered wavy and it is kept at a uniform hot temperature, so:
   \[
   \text{at} X = 1 \quad \theta = 1, \quad \text{and} \quad U = V = 0
   \]

**3. SOLUTION ALGORITHM**

The differential equations, represented by equations (2) to (5), together with respective boundary conditions are solved using the finite volume method described in Ferziger and Peric (1999). The solution domain is first subdivided into finite number of control volumes (CV). Body fitted, non-orthogonal grids are used. Grids are oriented in such a way that the number of CV is higher near the walls where higher gradients of variable values are expected (see Figure 2). A collocated variable arrangement is used in the present investigation, and the details can be found in the text by Ferziger and Peric (1999). All variables are calculated at the center of each CV. Central differencing is used to discretize the diffusion terms, whereas a blending of upwind and central differencing is used for the convection terms. The source terms in the governing transport equations are not functions of the respective transported variables and are calculated explicitly. The convective fluxes at the control volume faces are calculated as \( F = F_L + B_f \left( F^H - F^L \right)^\text{old} \), where the superscripts \( L \) and \( H \) imply that the fluxes are calculated by the lower-order upwind differencing and higher-order central differencing schemes, respectively. The value of the blending factor, \( B_f \), on the right-hand side ranges between 0 and 1, and the superscript “old” indicates the value at the previous iteration level, which is calculated explicitly and added to the source terms. The pressure–velocity coupling is achieved using the well-known SIMPLE method of Patankar (1980). Linear interpolation and numerical differentiation are used to express the cell-face value of the variables and their derivatives through the nodal values. The final discretized form of governing equations is solved iteratively using Stone’s SIP solver (Stone, 1968). Iteration is continued until difference between two consecutive field values of variables is less than or equal to \(10^{-6}\). For further stabilization of numerical algorithm, under relaxation factors of 0.1–0.7 are used.

**4. VALIDATION OF NUMERICAL RESULTS**

The present numerical algorithm is tested by investigating the same problem considered by Al-Amiri et al. (2007) using different flow conditions and geometries, which were reported for laminar mixed convection heat transfer in a lid-driven cavity which is heated from the wavy bottom wall using the same boundary conditions but the numerical scheme is different. The comparison is made using the following dimensionless parameters: \( Re = 500 \) and 1000, \( Pr = 0.71 \) and 1.0, \( Ri = 0.01 \) and 0.4, \( Gr = 10^4 \) and \( 10^5 \) and \( A = 0.05 \) and 1.0. Excellent agreement is achieved between Al-Amiri et al. (2007) and the present numerical scheme for both the streamlines and temperature contours inside the cavity as shown in Figure (3). These validations make a good confidence in the present numerical model to deal with the air-filled cavity with a wavy right side hot wall.

**5. RESULTS AND DISCUSSION**

The flow and thermal fields in a laminar square cavity which is heated from wavy right side wall while the cold left side wall is lid-driven are discussed by considering the effects of the Richardson number \( Ri \), undulations number and wavy right side surface amplitude. In the present work, the following data are taken into account: Richardson number \( (Ri) \) 0.01,1.0 and 10, Grashof number \( (Gr) = 10^4 \), wavy right side surface amplitude \( (A) = 0, 0.025, 0.05 \) and 0.075.

**5.1 Effect of Richardson Number**

Figures (4-6) explain the effect of the Richardson number on the stream contours and isotherms with no, one, two and three number of undulations with wavy right side wall amplitude equals 0.05. The Richardson number is ranged between 0.01 and 10. For very low Richardson number, the stream contours and isotherms are represented by major rotating vortices that can be observed in the center of the cavity. This case is identical with classical lid-driven flow. From the other hand, secondary vortices can be observed near the side edges of the cavity. Also, a clear increase in the isotherms near the right side hot wavy wall can be detected as shown in Figure (4) referring a thermal boundary layer can be found near this wall.

From the other hand, the temperature gradients effect is small in another cavity zones. The reason of this behaviour, is that the lid-driven cavity flow effect becomes dominate in these regions. In the case of the Richardson number becomes unity, as shown in Figure (5), the lid-driven effect in the left cold side wall equals the effect of the buoyancy forces which are generated due to natural convection and the heat transfer mechanism in this case is the mixed convection. In Figure (6), in the case of the Richardson number becomes 10, the buoyancy forces effect becomes greater than the lid-driven effect on the left side cold wall.
Figure 3 Comparison of the streamlines and temperature contours between the present work and that of Al-Amiri et al. (2007) using different flow conditions and geometries.

Present Results

Al-Amiri et al. (2007) Results
Figure 4 Streamlines (left) and isotherms (right) for various undulations ($Gr = 10^4$, $Ri = 0.01$, $A = 0.05$)
Figure 5 Streamlines (left) and isotherms (right) for various undulations ($Gr = 10^4$, $Ri = 1.0$, $A = 0.05$).
Figure 6 Streamlines (left) and isotherms (right) for various undulations (Gr = $10^4$, Ri = 10.0, A = 0.05).
Figure 7 Streamlines (left) and isotherms (right) for various amplitudes ($Gr = 10^4$, $Ri = 1.0$).
Figure 8 Local heat flux variation along the heated wavy wall for various numbers of undulations
\( (A=0.05, \ Gr = 10^4, \ Ri = 0.01, \ Pr = 0.71) \).

Figure 9 Local heat flux variation along the heated wavy wall for various numbers of undulations
\( (A=0.05, \ Gr = 10^4, \ Ri = 1.0, \ Pr = 0.71) \).

Figure 10 Local heat flux variation along the heated wavy wall for various numbers of undulations
\( (A=0.05, \ Gr = 10^4, \ Ri = 10.0, \ Pr = 0.71) \).
Figure 11 Local Nusselt number variations along the heated wavy wall for various numbers of undulations
\(A = 0.05, Gr = 10^4, Ri = 0.01, Pr = 0.71\)

Figure 12 Local Nusselt number variations along the heated wavy wall for various amplitudes
\(\lambda = 3, Gr = 10^4, Pr = 0.71, Ri = 0.01\)

Figure 13 Effects of Richardson number and the numbers of undulations on the average Nusselt number along the heated wavy wall at \(A = 0.05, Gr = 10^4, Pr = 0.71\)
5.2 Effect of Wavy Right Wall Amplitude
Figure (7) explains the wavy right wall amplitude effect on the flow and thermal fields when Richardson number is unity. The figure shows that both streamlines and temperature contours are changed slightly with increasing of undulations number. Also, for very low Richardson number, the heat flux has a maximum value. The reason of this behaviour is due to the increase in the velocity related with the mechanism of forced convection in this domain of Richardson number. Moreover, it can be observed that near the hot wavy side wall, the stream contours take the wavy shape of the wall.

5.3 Effect of Number of Undulations on the Local Heat Flux

Figures (8-10) show the undulations number effect on the local heat flux in the right side hot wavy wall. These figures show that the stream contours and isotherms do not change clearly with the increasing of undulations number. Also, for very low Richardson number, the heat flux has a maximum value. The reason of this phenomena is that when the Richardson number value is high causes a small effect in the convection heat transfer leading to make average Nusselt number is invariant. When the Richardson number is 0.01, the average Nusselt number high and different.

5.4 Effect of Undulations Number on the Local and Average Nusselt Number

Figure (11) explains variation of the local Nusselt number in various undulations numbers for very low Richardson number. It has been noticed at number of undulation equals two; the local Nusselt number reaches a maximum value which indicates a great heat flux in this region. This is due to increase in the speed of lid-driven.

5.5 Effect of Amplitude of Wavy Right Wall on The Local Nusselt Number

Figure (12) explains the effect of the amplitude of wavy right wall on the local Nusselt number. The results show that when amplitude of the wavy right wall is increased, a high local Nusselt number is observed. This is due to the increase in the heat transfer rate related to increase in velocity near the left lid-driven side wall.

5.6 Effect of Number of Undulations on The Average Nusselt Number

Figure (13) describes the undulations number (λ) effect for different Richardson numbers on the average Nusselt number. From this figure, it can be observed that as the Richardson numbers decrease, the average Nusselt number begins to increase. However, it can be noticed that the average Nusselt number is invariant when the Richardson number becomes 10. The reason of this phenomena is that when the Richardson number value is high causes a small effect in the convection heat transfer leading to make average Nusselt number is invariant. When the Richardson number is 0.01, the average Nusselt number has a high value and consequently a large effect in the convection heat transfer leading to make average Nusselt number high and different.

5.7 Effect of Wavy Wall Amplitude on the Local Heat Flux

Figures (14-16) explain the effect of varying the amplitude of the wavy wall on the local heat flux which is predicted at the right side wavy wall when Richardson number values are varied. The results show that when the amplitude of wavy wall is increased, the local heat flux is found to increase too. Also, the figures predict that the heat flux is found to have a large value for very low Richardson number. This is due to the forced convection effect occurred due to lid-driven effect in the left side wall.

6. CONCLUSIONS

The conclusions can be drawn from the results of the present work as follows. A high temperature gradients are observed near the right side hot wavy wall, which causes an increase in the convection heat transfer. While, in the another regions of the cavity the temperature differences are very slight and as a result, the temperature gradients are small due to the important effect of the lid-driven. For very low Richardson number, the streamlines and temperature contours are represented by major rotating vortices which can be observed in the center of the cavity. It can be noticed that the average Nusselt number remains invariant when the Richardson number becomes high. A high local heat flux is observed at low Richardson number while, high local Nusselt number is predicted with the increase in the amplitude of the wavy wall. Both the flow and thermal fields are change slightly with surface amplitude in the considered range of wavy right side wall amplitude. It has been observed that the local Nusselt number has a maximum value at λ = 2 which refers a high local heat flux due to increase in the speed of left lid-driven side wall. Except near the wavy side wall, the variation of undulations number not affects the flow and thermal fields. The local heat flux is found to increase with the increase in the amplitude value of the wavy hot right side wall. The results of the present study indicate that the wavy cavity with lid-driven can be considered as a significant heat transfer tool at high wavy wall amplitudes and low Richardson number.

REFERENCES


